INSTRUCTOR'S MANUAL

To Accompany

# ENGINEERING MECHANICS - DYNAMICS

Volume 2

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## USE OF THE INSTRUCTOR'S MANUAL

The problem solution portion of this manual has been prepared for the instructor who wishes to occasionally refer to the authors' method of solution or who wishes to check the answer of his (her) solution with the result obtained by the authors. In the interest of space and the associated cost of educational materials, the solutions are very concise. Because the problem solution material is not intended for posting of solutions or classroom presentation, the authors request that it not be used for these purposes.

In the transparency master section there are approximately 65 solved problems selected to illustrate typical applications. These problems are different from and in addition to those in the textbook. Instructors who have adopted the textbook are granted permission to reproduce these masters for classroom use.

2/1

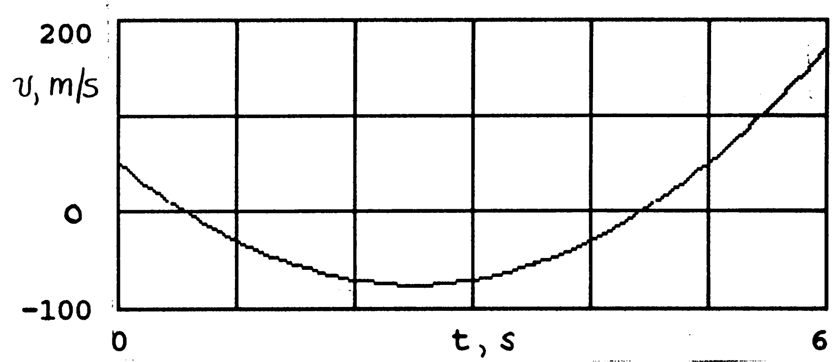
$v = 20t^2 - 100t + 50$

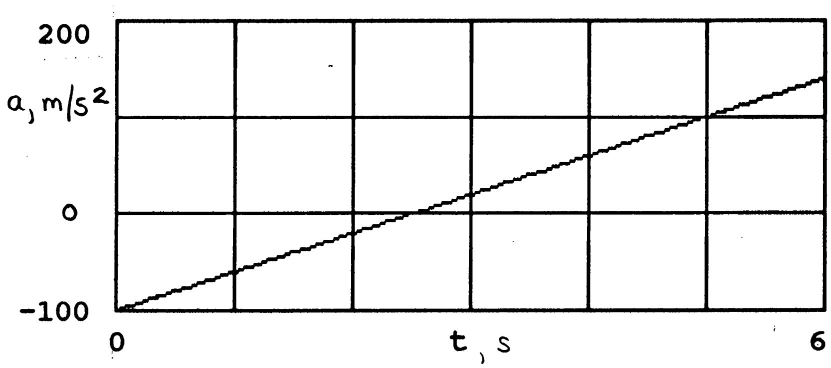
$a = \frac{dv}{dt} = \underline{40t - 100}$ } See plots

$a = 0: \quad 40t - 100 = 0, \quad t = 2.5 \text{ s}$

At $t = 2.5$ s, $v = 20(2.5)^2 - 100(2.5) + 50$

$= \underline{-75 \text{ m/s}}$



**2/2**

$$s = 2t^3 - 30t^2 + 100t - 50$$

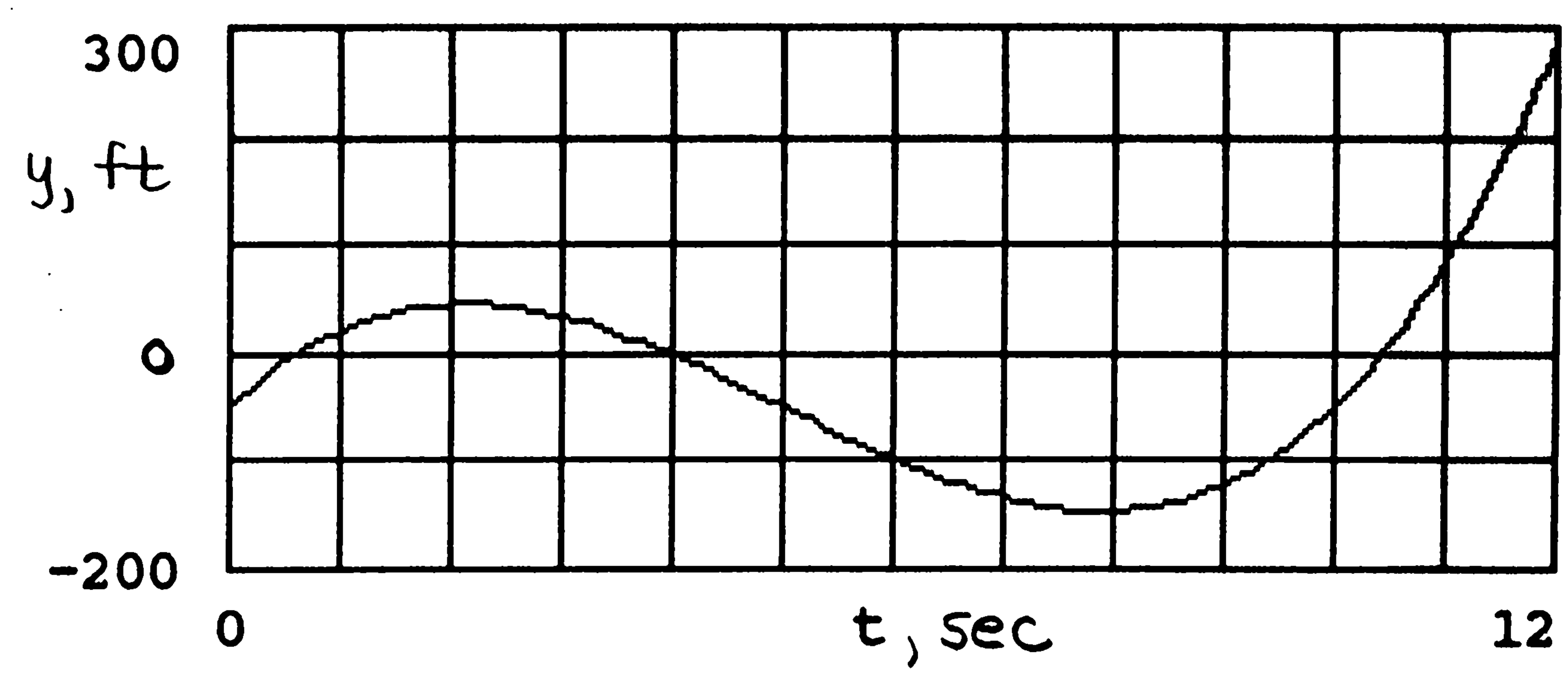
$$v = \frac{ds}{dt} = 6t^2 - 60t + 100$$

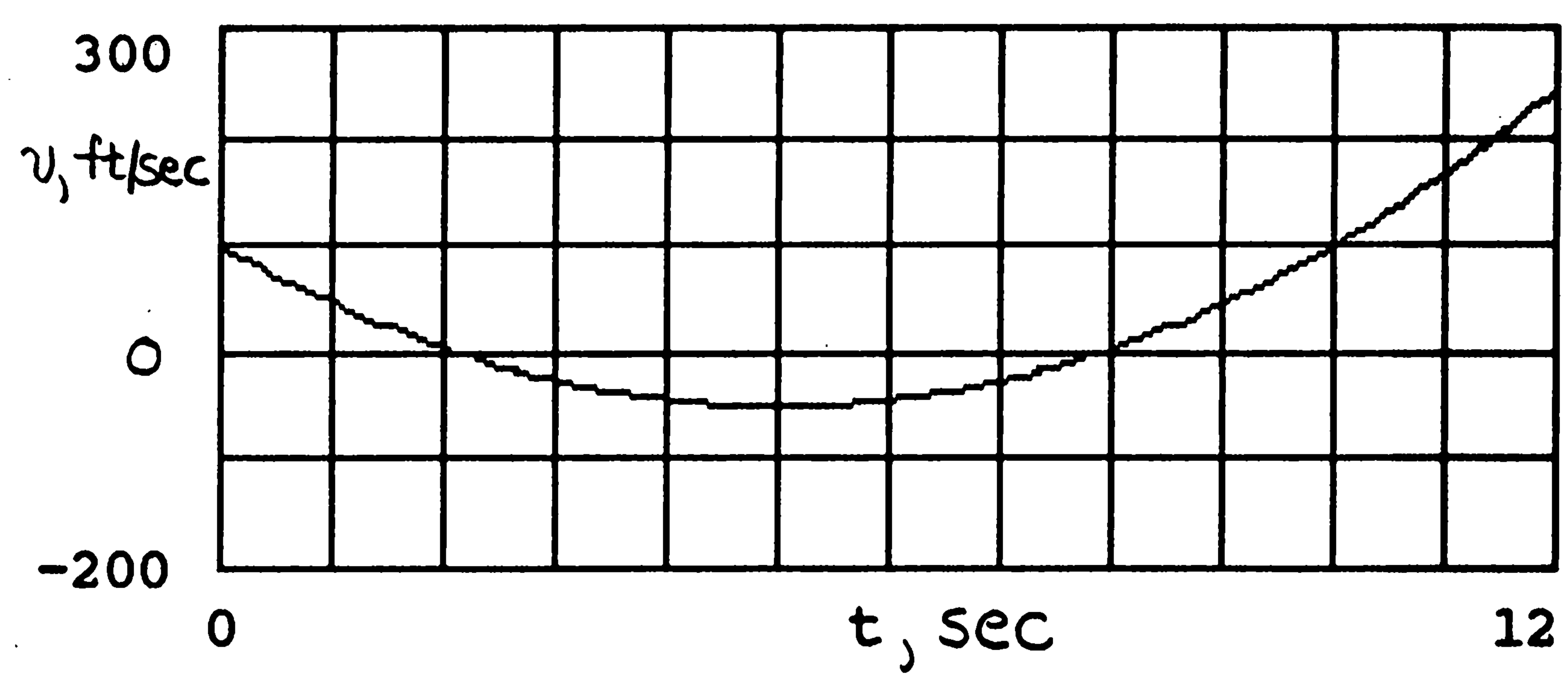
$$a = \frac{dv}{dt} = 12t - 60$$

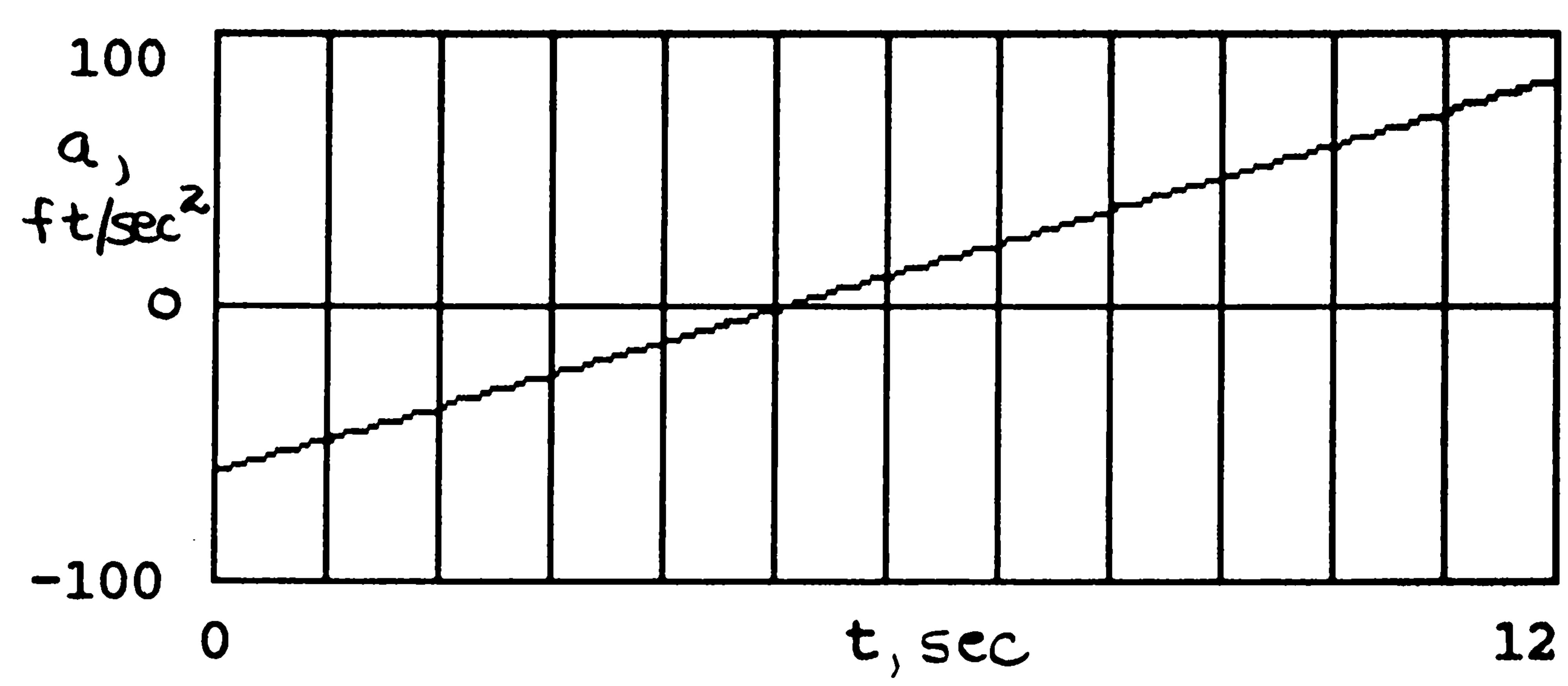
See plots

$v = 0$ : $6t^2 - 60t + 100 = 0$

$$t = \frac{60 \pm \sqrt{60^2 - 4(6)(100)}}{2 \cdot 6} = \underline{2.11 \text{ sec}, \; 7.89 \text{ sec}}$$



$\boxed{2/3}$ $v = 2 + 5t^{3/2}$

$$a = \frac{dv}{dt} = \frac{3}{2} \cdot 5t^{\frac{1}{2}} = \frac{15}{2}\sqrt{t}$$

$$\frac{ds}{dt} = 2 + 5t^{3/2}$$

$$\int_{s_0=0}^{s} ds = \int_0^t \left(2 + 5t^{3/2}\right) dt$$

$$s = 2t + 2t^{5/2}$$

At $t = 4\,s$ : $\begin{cases} s = 72 \text{ m}, \quad v = 42 \text{ m/s} \\ a = 15 \text{ m/s}^2 \end{cases}$

**2/4** $v = 5s^{3/2}$

$$a = \frac{dv}{dt} = \frac{3}{2}5s^{1/2}\frac{ds}{dt} = \frac{15}{2}s^{1/2}v$$

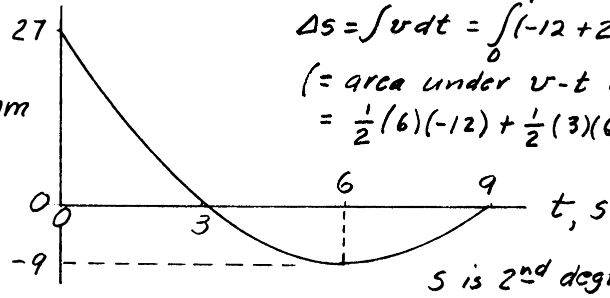
$$= \frac{15}{2}s^{1/2}\left(5s^{3/2}\right) = \frac{75}{2}s^2$$

When $s = 2$ mm, $a = \frac{75}{2}2^2 = \underline{150 \text{ mm/s}^2}$

**2/5** $s = 27 - 12t + t^2$ mm

$v = \dot{s} = -12 + 2t$ mm/s

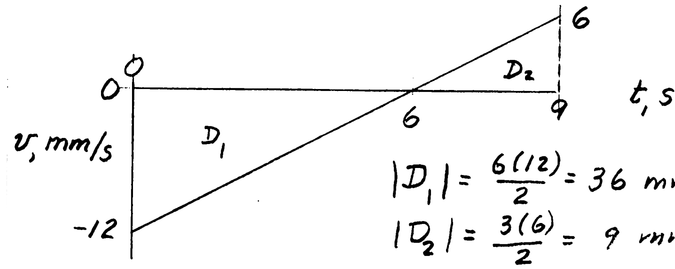
$a = \dot{v} = 2$ mm/s$^2$




$\Delta s = \int v\,dt = \int_0^9 (-12 + 2t)\,dt = \underline{-27 \text{ mm}}$

(= area under $v$-$t$ curve
$= \frac{1}{2}(6)(-12) + \frac{1}{2}(3)(6) = -27$ mm)

$s$ is 2nd degree in $t$ so
$a = \ddot{s} = \underline{\text{constant}}$




$|D_1| = \frac{6(12)}{2} = 36$ mm

$|D_2| = \frac{3(6)}{2} = 9$ mm

$D = 36 + 9 = \underline{45 \text{ mm}}$

2/6 $v = \frac{ds}{dt} = 400 - 16t^2$

$$\Delta s = \int_0^{\Delta s} ds = \int_0^6 (400 - 16t^2)\,dt, \quad \Delta s = 400t - \frac{16}{3}t^3 \Big]_0^6 =$$

$$= 2400 - 1152 = 1248 \text{ mm}$$

$$\Delta s = \underline{1.248 \text{ m}}$$

$$D = \Delta s_1 + |\Delta s_2|, \quad \Delta s_1 = \int_0^5 (400 - 16t^2)\,dt = 1333.3 \text{ mm}$$

$$|\Delta s_2| = \left| \int_5^6 (400 - 16t^2)\,dt \right| = 85.3 \text{ mm}$$

$$D = 1333.3 + 85.3 = 1418.7 \text{ mm}$$

or $\underline{D = 1.419 \text{ m}}$

2/7

$$a = \frac{dv}{dt} = 4t - 30$$

$$\int_{v_0 = 3}^{v} dv = \int_0^t (4t - 30)\, dt, \quad \underline{v = 3 - 30t + 2t^2 \text{ m/s}}$$

$$\frac{ds}{dt} = 3 + 2t^2 - 30t$$

$$\int_{s_0 = -5}^{s} ds = \int_0^t (3 + 2t^2 - 30t)\, dt$$

$$\underline{s = -5 + 3t - 15t^2 + \frac{2}{3}t^3 \text{ m}}$$

2/8 For constant acceleration,

$$s = \frac{1}{2}at^2, \quad t = \left(\frac{2s}{a}\right)^{1/2} = \left(\frac{2(30\,000)}{1.5(9.81)}\right)^{1/2}$$

$$= \underline{63.9 \text{ s}}$$

$$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30\,000)} = \underline{940 \text{ m/s}}$$

2/9

$$v^2 - v_0^2 = 2a(s - s_0)$$

$$0 - \left[50\ \frac{5280}{3600}\right]^2 = 2a(100),\quad a = -26.9\ \frac{\text{ft}}{\text{sec}^2}$$

Then $$0 - \left[70\ \frac{5280}{3600}\right]^2 = 2(-26.9)\,s$$

$$\underline{s = 196.0 \text{ ft}}$$

$\boxed{2/10}$ For $a =$ constant, $v^2 = v_0^2 + 2as$

$$\left[\frac{180\,(5280)}{3600}\right]^2 = 0^2 + 2a\,(300)$$

$$a = 116.2 \text{ ft/sec}^2$$

or $a = 116.2/32.2 = \underline{3.61g}$

$\boxed{2/11}$ $v^2 = v_0^2 + 2a(s - s_0)$

$$\left(\frac{200}{3.6}\right)^2 = 0^2 + 2(0.4 \cdot 9.81)\,s$$

$$\underline{s = 393 \text{ m}}$$

$v = v_0 + at$ : $\left(\frac{200}{3.6}\right) = 0 + 0.4(9.81)\,t$

$$\underline{t = 14.16 \text{ s}}$$

**2/12** $\int v\,dv = \int a\,ds$ ; $\int_{200/3.6}^{30/3.6} v\,dv = a\int_0^{600} ds$

200 km/h
$= 200/3.6$ m/s

30 km/h
$= 30/3.6$ m/s

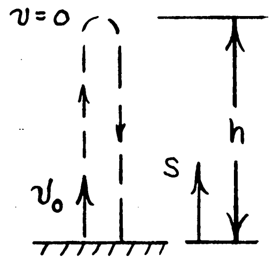
$$\frac{1}{2(3.6)^2}(\overline{30}^2 - \overline{200}^2) = 600a$$

$$\underline{a = -2.51 \text{ m/s}^2}$$

$\underline{2/13}$ $v^2 = v_0^2 + 2as$ , where $a = g/6$

$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5$ , $\underline{v = 4.51 \text{ m/s}}$

**2/14**




$$v^2 = v_0^2 + 2a(s - s_0)$$

Apex: $0^2 = 200^2 + 2(-9.81)\,h$

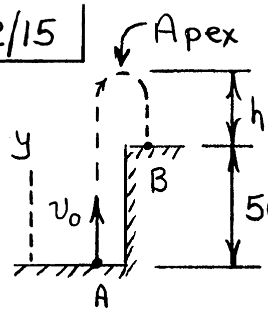
$$\underline{h = 2040 \text{ m}}$$

$$v = v_0 + at$$

Impact : $-200 = 200 - 9.81t$

$$\underline{t = 40.8 \text{ s}}$$

2/15




Evaluate $v^2 = v_0^2 - 2g(y - y_0)$ at apex:

$0 = 80^2 - 2(32.2)(50 + h - 0)$

$\underline{h = 49.4 \text{ ft}}$

Evaluate $y = y_0 + v_0 t - \frac{1}{2} g t^2$ at B:

$50 = 0 + 80t - 16.1t^2$, or $16.1t^2 - 80t + 50 = 0$

$$t = \frac{80 \pm \sqrt{80^2 - 4(16.1)(50)}}{2(16.1)} = 0.733,\ 4.26 \text{ sec}$$

$\underline{t = 4.24 \text{ sec}}$ represents the <u>second</u> time at which $y = 50$ ft.

$v_B = v_0 - gt = 80 - 32.2(4.24) = -56.4$ ft/sec

(or <u>56.4 ft/sec downward</u>)

2/16 Acceleration period:

$v = v_0 + at$ : $\frac{22}{3.6} = 0 + \frac{9.81}{4} t_a$, $t_a = 2.49$ s

Note that the deceleration time $t_d = t_a$

$v^2 = v_0^2 + 2a\Delta s$ : $\left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\,\Delta s_a$

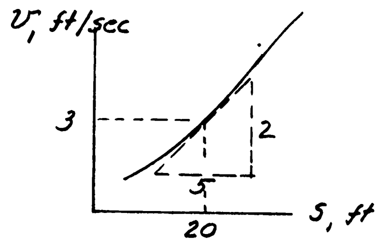
$\Delta s_a = 7.61$ m $= \Delta s_d$

Cruise period: $\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335$ m

$\Delta s = v_c t_c$ : $335 = \frac{22}{3.6} t_c$, $t_c = 54.8$ s

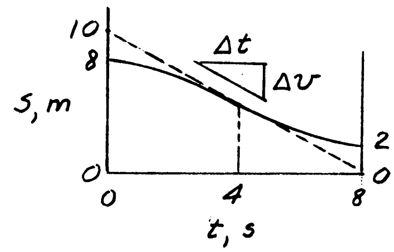
Total run time $t = t_c + t_a + t_d = \underline{59.8\text{ s}}$

2/17




$$a = v\frac{dv}{ds} = 3\left(\frac{2}{5}\right) = \underline{1.2 \text{ ft/sec}^2}$$

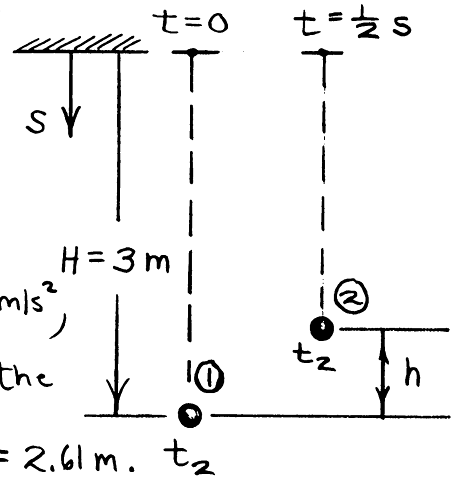
2/18




$$v_{AV} = \frac{s_2 - s_1}{\Delta t} = \frac{2-8}{8} = \underline{-0.75 \text{ m/s}}$$

$$v = v_4 = \frac{ds}{dt} = \frac{-10}{8} = \underline{-1.25 \text{ m/s}}$$

2/19 $s = s_0 + v_0 t + \frac{1}{2} g t^2$




Sphere ①: $H = \frac{1}{2} g t_2^2$

Sphere ②:

$$(H-h) = \frac{1}{2} g \left(t_2 - \frac{1}{2}\right)^2$$

With $H = 3$ m & $g = 9.81$ m/s$^2$, eliminate $t_2$ between the 2 equations & obtain $\underline{h = 2.61 \text{ m}}$.

$\underline{2/20}$ $v_C = v_B + a\Delta t_{B-C}$, $a = \dfrac{(60-100)/3.6}{4}$

$= -2.78 \text{ m/s}^2$

$$\Delta s_{B-C} = v_B \Delta t_{B-C} + \frac{1}{2} a \Delta t_{B-C}^2$$

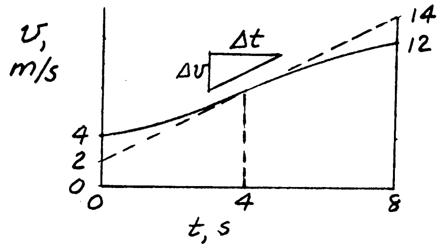
$$= \frac{100}{3.6} 4 + \frac{1}{2}(-2.78)4^2 = 88.9 \text{ m}$$

$$\Delta s_{A-D} = \Delta s_{A-B} + \Delta s_{B-C} + \Delta s_{C-D}$$

$$3000 = \frac{100}{3.6} t + 88.9 + \frac{60}{3.6} t, \quad \underline{t = 65.5 \text{ s}}$$

$$s = \Delta s_{A-B} = \frac{100}{3.6}(65.5) = 1819 \text{ m} \quad \text{or} \quad \underline{s = 1.819 \text{ km}}$$

2/21




$a_{AV} = \frac{\Delta v}{\Delta t} = \frac{12-4}{8} = 1.00\ m/s^2$

$a_4 = \frac{dv}{dt} = \frac{14-2}{8} = 1.50\ m/s^2$

$\Delta a = a_4 - a_{AV} = 1.50 - 1.00 = \underline{0.50\ \frac{m}{s^2}}$

$\Delta s = \int v\,dt \approx \frac{14+2}{2}(8) = \underline{64\ m}$

**2/22** $\int_{0.4}^{v} v\,dv = \int a_x dx$ ; $\frac{1}{2}(v^2 - 0.4^2) =$ area under $a_x$-$x$ curve

Area $= \int a_x dx = (a_x)_{av} \Delta x = 3(120-40)10^{-3} = 0.240 (m/s)^2$

Thus $v^2 = 0.4^2 + 2(0.240) = 0.16 + 0.48 = 0.64$

$v = \sqrt{0.64} = \underline{0.8 \text{ m/s}}$

$\boxed{2/23}$ $v^2 = v_0^2 + 2a(s - s_0)$

$0 = 4^2 + 2\left(-\frac{9.81}{4}\right)(s)$ , $\underline{s = 3.26 \text{ m}}$

$v = v_0 + at$ : $0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}$ , $t_{up} = 1.631 \text{ s}$

$t = 2t_{up} = 2(1.631) = \underline{3.26 \text{ s}}$

**2/24** $a = 400 - kx$, where $k = \frac{400}{6/12}$ sec$^{-2}$

$$a = 400(1 - 2x) \quad (x \text{ in ft})$$

$v\,dv = a\,dx$ : $\int_0^v v\,dv = 400\int_0^x (1-2x)\,dx$

$v^2 = 800(x - x^2)$, $v = \frac{dx}{dt} = 20\sqrt{2}\sqrt{x-x^2}$

(taking + sign)

$$\int_0^t dt = \int_0^x \frac{dx}{20\sqrt{2}\sqrt{x-x^2}}$$

$$t = -\frac{1}{20\sqrt{2}} \sin^{-1} \frac{1-2x}{\sqrt{1}} \Big|_0^x = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \sin^{-1}(1-2x)\right]$$

(a) $x = \frac{1}{4}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right] = \underline{0.0370 \text{ sec}}$

(b) $x = \frac{1}{2}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - 0\right] = \underline{\underline{0.0555 \text{ sec}}}$

2/25 $v_0 = 100/3.6 = 27.8 \text{ m/s}$

$a = -g \sin\theta = -9.81 \sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588 \text{ m/s}^2$

(a) $v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$

(b) $v^2 = v_0^2 + 2a(s - s_0) = 27.8^2 + 2(-0.588)(100)$

$v = \underline{25.6 \text{ m/s}}$

2/26 Train: $v^2 = v_1^2 + 2as$ : $60^2 = 80^2 + 2a\left(\frac{1}{2}\right)$

$$a = -2800 \text{ mi/hr}^2$$

$s = v_1 t + \frac{1}{2}at^2$ : $1 = 80t - \frac{2800}{2}t^2$

$t = 0.01847$ hr or $t = 0.03867$ hr (disregard)

So $t = 0.01847(60)^2 = 66.5$ sec

Car: $t = 66.5 - 4 = 62.5$ sec

$s = v_1 t + \frac{1}{2}at^2$ : $1.3(5280) = 50\frac{44}{30}(62.5) + \frac{a}{2}(62.5)^2$

$\underline{a = 1.168 \text{ ft/sec}^2}$

$v = v_1 + at = 50 + 1.168 \frac{3600^2}{5280}\frac{62.5}{3600} = \underline{99.8 \frac{\text{mi}}{\text{hr}}}$

2/27 For constant accel.,

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at$$

when $t = 0$, $x_0 = 4$ m, $v_0 = 3$ m/s

$t = 4$ s, $\dot{x} = 0$

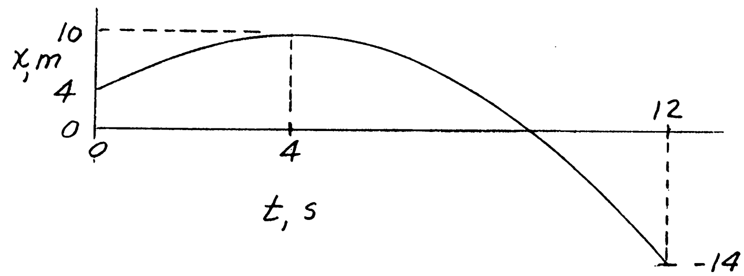
Thus $x = 4 + 3t + \frac{1}{2} a t^2$

Also $0 = 3 + 4a$, $a = -3/4$ m/s$^2$

so $x = 4 + 3t - \frac{3}{8} t^2$

when $t = 12$ s, $x = 4 + 3(12) - \frac{3}{8}(12^2) = \underline{-14 \text{ m}}$

" $t = 4$ s, $x = x_{max} = 4 + 3(4) - \frac{3}{8}(4^2) = \underline{10 \text{ m}}$



**2/28** $a = \frac{dv}{dt}$ ; $v_m = \int a\,dt = at = 6(20) = \underline{120\text{ m/s}}$

Corresponding $h = \frac{1}{2}at^2 = \frac{1}{2}(6)(20)^2 = 1200\text{ m}$

During upward coast, $\int_{v_m}^{0} v\,dv = \int_{0}^{\Delta h} -g\,dy$

$v_m^{\,2} = 2g\Delta h$, $\Delta h = \frac{\overline{120}^2}{2(9.81)} = 734\text{ m}$

Max. $h = 1200 + 734 = 1934\text{ m}$ or $\underline{h = 1.934\text{ km}}$

$\boxed{2/29}$ $s_{car} = vt = \frac{120}{3.6} t$

$s_{cycle} = v_{av} t_1 + v_{max} t_2 = \frac{1}{2} \frac{150}{3.6} t_1 + \frac{150}{3.6} t_2$

where $t_1 = \frac{v_{max}}{a} = \frac{150}{3.6 \times 6} = 6.94 \text{ s}$ & $t_2 = t - 6.94 - 2$

$s_{car} = s_{cycle}$ ; $\frac{120}{3.6} t = \frac{75}{3.6} 6.94 + \frac{150}{3.6} (t - 8.94)$

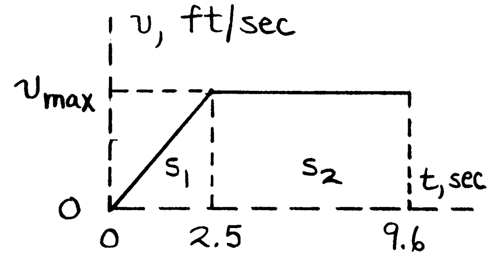
$30t = 820.8$ , $t = 27.36 \text{ s}$

$s = \frac{120}{3.6} (27.36) = \underline{912 \text{ m}}$

2/30

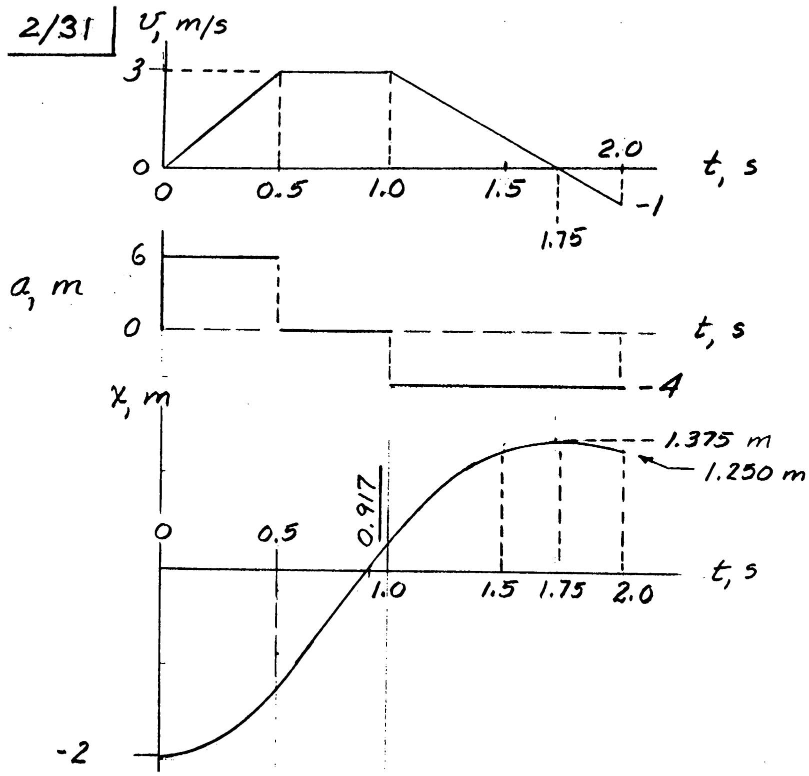
$$s_1 = \frac{1}{2}(2.5)v_{max}$$

$$s_2 = (9.6-2.5)v_{max}$$

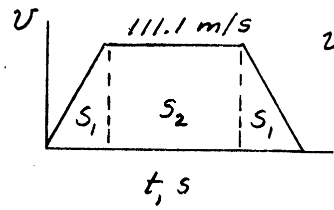



$$s_1 + s_2 = (1.25 + 7.1)v_{max} = 100(3)$$

$$\underline{v_{max} = 35.9 \text{ ft/sec}}$$


2/31
v, m/s
3
0
0
0.5
1.0
1.5
2.0
t, s
1.75
-1
a, m
6
0
t, s
-4
x, m
1.375 m
1.250 m
0.917
0
0.5
1.0
1.5
1.75
2.0
t, s
-2

2/32 | 400 km/h $\equiv \frac{400}{3.6} = 111.1$ m/s




$v^2 = 2as$, $s_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049$ m

$s_2 = 10\,000 - 2(1049)$
$= 7903$ m

$t_1 = \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88$ s

$t_2 = \frac{s_2}{v} = \frac{7903}{111.1} = 71.13$ s

$t = 2t_1 + t_2$
$= 2(18.88) + 71.13$
$= \underline{108.9 \text{ s}}$

or $\underline{t = 1.81 \text{ min}}$

**2/33** $v^2 = \frac{k}{s}$, $v = \dot{s} = \sqrt{k/s}$ where $k = 2^2(9) = 36$ $in^3/sec^2$

$\int_9^s \sqrt{s}\, ds = 6\int_0^t dt$, $\frac{2}{3} s^{3/2}\Big]_9^s = 6(t-0)$,

$s^{3/2} = 27 + 9t$, but $v = \sqrt{k}\, s^{-1/2}$ so

$v = 6(27+9t)^{-1/3}$ & at $t = 3$ sec, $v = 6(27+27)^{-1/3}$

$= 2^{2/3} = \underline{1.587 \frac{in.}{sec}}$

2/34 Particle 1 : $a = -kv$

$$-kv = \frac{dv}{dt}$$

$$-k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$$\int_{s_0=0}^{s} ds = v_0 \int_{t_0=0}^{t} e^{-kt} dt \Rightarrow \underline{s = \frac{v_0}{k}\left(1 - e^{-kt}\right)}$$

Particle 2 : $a = -kt$

$$-kt = \frac{dv}{dt}$$

$$-k\int_0^t t\,dt = \int_{v_0}^{v} dv \Rightarrow \underline{v = v_0 - \tfrac{1}{2}kt^2}$$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$$\int_0^s ds = \int_0^t \left(v_0 - \tfrac{1}{2}kt^2\right) dt \Rightarrow \underline{s = v_0 t - \tfrac{1}{6}kt^3}$$

Particle 3 : $a = -ks$

$$-ks = v\frac{dv}{ds}$$

$$-k\int_0^s s\,ds = \int_{v_0}^{v} v\,dv \Rightarrow \underline{v = \pm\sqrt{v_0^2 - ks^2}}$$

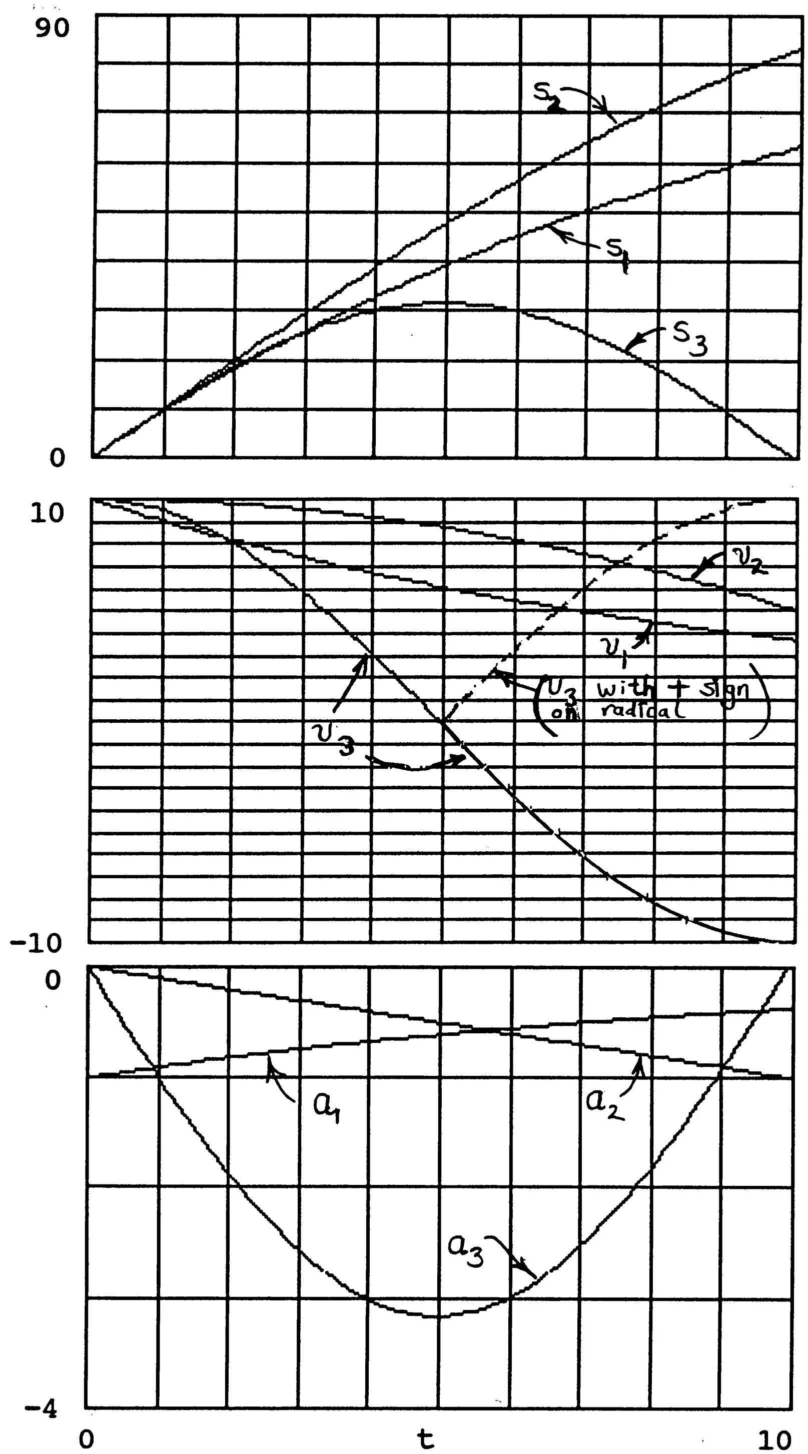
Note: Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign, etc.

Then

$$\frac{ds}{dt} = \pm\sqrt{v_0^2 - ks^2}$$

$$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{k}} \sin^{-1}\left(\frac{\sqrt{k}}{v_0} s\right) = t \Rightarrow \underline{s = \frac{v_0}{\sqrt{k}} \sin\left(\sqrt{k}\,t\right)}$$



90
0
10
-10
0
-4
0
t
10
$s_2$
$s_1$
$s_3$
$v_2$
$v_1$
$v_3$
($v_3$ with + sign on radical)
$a_1$
$a_2$
$a_3$

2/35 $0 < t < 10\text{ s}: \quad a = 6 - kt, \quad k = \frac{6}{10}\text{ m/s}^3$

$a = \frac{dv}{dt} = 6\left(1 - \frac{t}{10}\right), \quad t \text{ in s}, \quad a \text{ in m/s}^2$

$\int_0^v dv = \int_0^t 6\left(1 - \frac{t}{10}\right) dt, \quad v = 6t - \frac{3}{10}t^2$

$v_{10} = 6(10) - \frac{3}{10}(10)^2 = 30 \text{ m/s}$

$v = \frac{ds}{dt}, \quad s_{10} = \int_0^{10}\left(6t - \frac{3}{10}t^2\right) dt = 200 \text{ m}$

$t > 10\text{ s}: \quad \Delta s = v_{10}\,\Delta t, \quad \Delta t = \frac{400 - 200}{30} = 6.67\text{ s}$

$t = 10 + \Delta t = \underline{16.67\text{ s}}$

2/36

$$a = g - cy = v\frac{dv}{dy}$$

$$\int_0^{y_m}(g - cy)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^2}{2}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^2}{2} = -\frac{v_0^2}{2} \Rightarrow \underline{c = \frac{v_0^2 + 2gy_m}{y_m^2}}$$

2/37 $v\,dv = a\,ds$; $\frac{v\,dv}{-Kv^2} = ds$, $\int_{v_1}^{v_2} \frac{dv}{v} = -K\int_0^s ds$

$\ln\frac{v_2}{v_1} = -Ks$, $K = \frac{1}{s}\ln\frac{v_1}{v_2} = \frac{1}{1500}\ln\frac{100}{20} = \underline{1.073(10^{-3})\text{ ft}^{-1}}$

$a = \frac{dv}{dt}$; $-Kv^2 = \frac{dv}{dt}$, $\int_{v_1}^{v_2}\frac{dv}{v^2} = -Kt$, $t = \frac{1}{K}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$

$t = \frac{10^3}{1.073}\left(\frac{1}{20} - \frac{1}{100}\right)\frac{30}{44} = \underline{25.4\text{ sec}}$

**2/38** $v\,dv = a\,dx$, $\int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{-C_1 - C_2 v^2}$

$$x = \frac{-1}{2C_2}\ln(+C_1 + C_2 v^2)\Big]_{v_0}^{v} = \frac{1}{2C_2}\ln\frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

when $v = 0$, $\underline{x = D = \frac{1}{2C_2}\ln\left(1 + \frac{C_2}{C_1} v_0^2\right)}$

$\underline{2/39}$ (a) $g_0 = 32.2 \text{ ft/sec}^2 = \text{constant}$

$$v^2 = v_0^2 + 2a(s - s_0) : v^2 = 0^2 + 2(32.2)(500 \cdot 5280)$$

$$v = \underline{13{,}040 \text{ ft/sec}}$$

(b) $a = -g_0 \dfrac{R^2}{r^2} = v \dfrac{dv}{dr}$

$$-g_0 R^2 \int_{R+h}^{R} \frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-g_0 R^2 \left(-\frac{1}{r}\right)\Bigg|_{R+h}^{R} = \frac{1}{2} v^2 \Bigg|_0^{v}$$

$$\Rightarrow \quad v = \sqrt{\frac{2 g_0 R h}{R+h}} = \sqrt{\frac{2(32.2)(3959)(500)(5280)^2}{(3959+500)(5280)}}$$

$$= \underline{12{,}290 \text{ ft/sec}}$$

2/40

$$a = \frac{dv}{dt} = -kv, \quad \int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$$

$$\ln \frac{v}{v_0} = -kt, \quad \underline{v = v_0 e^{-kt}}$$

$$v = \frac{dx}{dt} = v_0 e^{-kt}, \quad \int_0^x dx = \int_0^t v_0 e^{-kt} dt$$

$$\underline{x = \frac{v_0}{k}\left[1 - e^{-kt}\right]}$$

$$v\,dv = a\,dx, \quad \frac{v\,dv}{v} = -k\,dx$$

$$\int_{v_0}^{v} dv = -k\int_0^x dx, \quad \underline{v = v_0 - kx}$$

2/41 (a) $a = 2 \text{ m/s}^2 = \text{constant}$

With $v = 250/3.6 = 69.4$ m/s, we have

$v^2 - v_0^2 = 2a(s - s_0)$ : $69.4^2 - 0^2 = 2(2)s$

$$\underline{s = 1206 \text{ m}}$$

(b) $a = a_0 - kv^2 = v\dfrac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k}\ln\left(a_0 - kv^2\right)\Big|_0^v$$

$$= -\frac{1}{2k}\ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$s = -\frac{1}{2(4)(10^{-5})}\ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right]$$

$$= \underline{1268 \text{ m}}$$

2/42 A to B:

$v_B^2 = v_A^2 + 2a\Delta s$ : $v_B^2 = 4^2 + 2(0.3)(32.2)(10)$

$v_B = 14.46$ ft/sec

$v_B = v_A + at$ : $14.46 = 4 + (0.3)(32.2)t_{AB}$

$t_{AB} = 1.083$ sec

B to C:

$v_C^2 = v_B^2 + 2a\Delta s$ : $0 = 14.46^2 + 2a(12)$

$\underline{a = -8.72 \text{ ft/sec}^2}$

$v_C = v_B + at$ : $0 = 14.46 - 8.72t_{BC}$

$t_{BC} = 1.659$ sec

$t = t_{AB} + t_{BC} = 1.083 + 1.659 = \underline{2.74 \text{ sec}}$

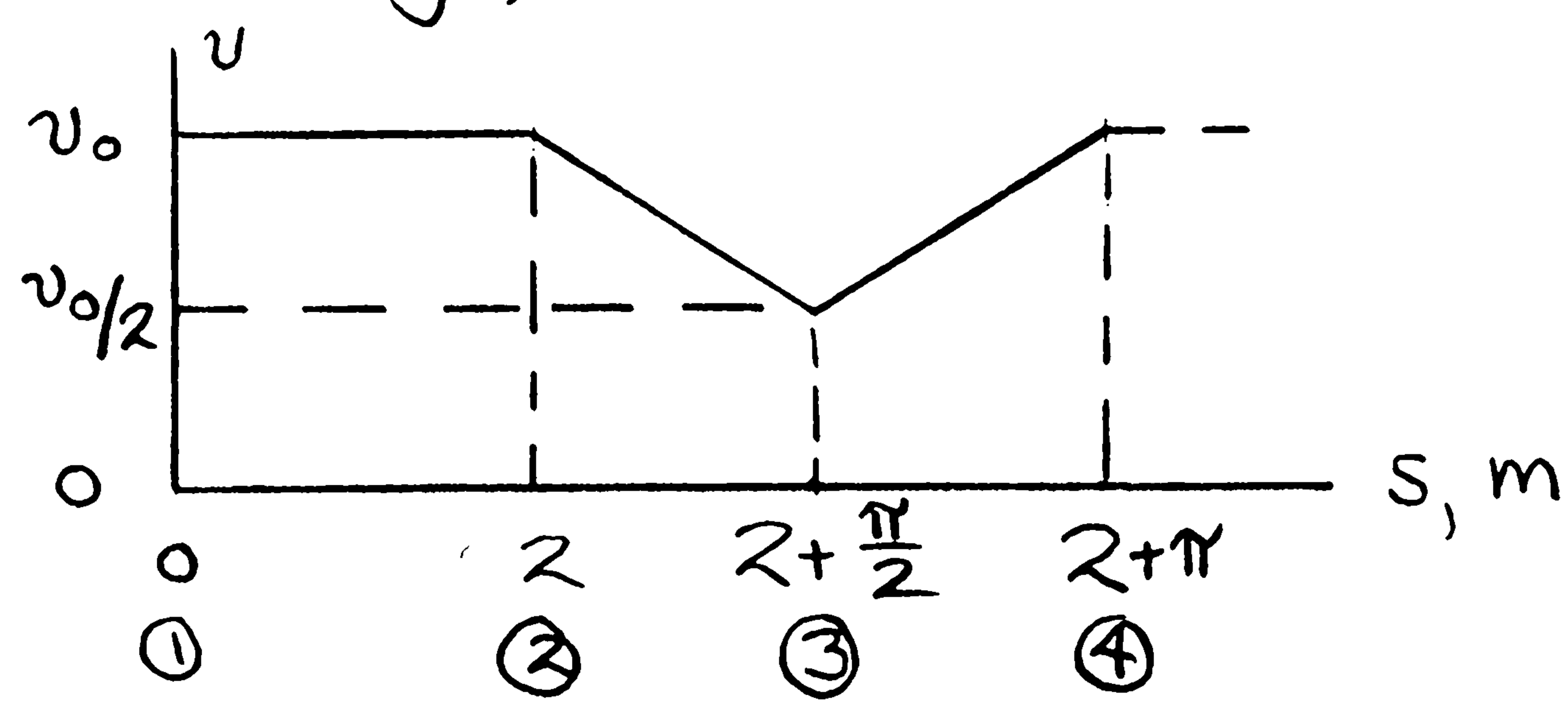
**2/43** $v\,dv = a\,dx$, $\int_0^v v\,dv = \int_0^x \frac{K}{(L-x)^2}\,dx$

$\frac{v^2}{2} = \left.\frac{K}{L-x}\right]_0^x$ ; $v^2 = \frac{2Kx}{L(L-x)}$

For $x = L - D/2$, $\underline{v = 2\sqrt{\frac{K(L-D/2)}{LD}}}$

2/44 Compute the time $t_{1/2}$ required for $\frac{1}{2}$ lap.

Temporarily, let $v_0$ be the straightaway speed:




Time $t_{1-2}$ to go from ① to ②: $t_{1-2} = \frac{2}{v_0}$

For $t_{2-3}$ : Note that $\frac{dv}{ds} = -c$ ($c$ = pos. const.)

so that $a_t = v\frac{dv}{ds} = -cv = \frac{dv}{dt}$

$$-c\int_{t_2}^{t_3} dt = \int_{v_2}^{v_3} \frac{dv}{v} \quad : \quad -ct_{2-3} = \ln\left(\frac{v_0/2}{v_0}\right)$$

$$t_{2-3} = \frac{1}{c}\ln 2$$

Likewise, $t_{3-4} = \frac{1}{c}\ln 2$

$c = \frac{v_0/2}{\pi/2} = \frac{v_0}{\pi}$, $t_{\frac{1}{2}} = t_{1-2} + t_{2-3} + t_{3-4} = \frac{6.36}{v_0}$

Lap time $t = 2t_{1/2} = \frac{12.71}{v_0} = \frac{12.71}{0.25} = \underline{50.8 \text{ s}}$

**2/45** 1st interval; $v_8 = at$, $v_8 = 1(8) = 8$ m/s

$\Delta t = 8$ s $\quad s_8 = \frac{1}{2}at^2$, $s_8 = \frac{1}{2}(1)(8^2) = 32$ m

2nd interval; $\Delta v = \int a\,dt$; $v_{14} - 8 = 2(6)$, $v_{14} = 20$ m/s

$\Delta t = 6$ s $\quad \Delta s = v_0 t + \frac{1}{2}at^2 = 8(6) + \frac{1}{2}(2)(6^2) = 84$ m

$s_{14} - 32 = 84$, $s_{14} = 116$ m

3rd interval; $\Delta s = vt$, $s_{24} - 116 = 20(10)$, $s_{24} = 316$ m

$\Delta t = 10$ s

Total interval; $\Delta v = \int a\,dt$, $0 - 0 = 8 + 12 + 0 - 2\Delta t$

$\underline{\Delta t = 10 \text{ s}}$

4th interval; $\Delta s = v_0 t + \frac{1}{2}at^2 = 20(10) + \frac{1}{2}(-2)(10)^2$

$s_{34} - s_{24} = 100$ m

$s_{34} = s = 100 + 316 = \underline{416 \text{ m}}$

**2/46** $a = -kv^2$,

$v\,dv = a\,dx$ , $\int_{v_0}^{v} \frac{v\,dv}{-kv^2} = \int_0^x dx$ , $x = \frac{-1}{k} \ln v \Big]_{v_0}^{v}$

$x = \frac{1}{k} \ln v_0/v$

when $v = v_0/2$, $x = D = \frac{1}{k} \ln 2 = \underline{0.693/k}$

$v = \frac{dx}{dt}$ where $kx = \ln v_0/v$, $v = v_0 e^{-kx}$

so $\frac{dx}{v_0 e^{-kx}} = dt$ or $\int_0^t dt = \frac{1}{v_0} \int_0^x e^{kx}\,dx$

& $t = \frac{1}{v_0} \frac{1}{k} e^{kx} \Big]_0^x = \frac{1}{kv_0} \left[ e^{kx} - 1 \right]$

For $x = D$, $e^{kx} = 2$ so $t = \frac{1}{kv_0} [2 - 1]$, $\underline{t = \frac{1}{kv_0}}$

$\underline{2/47}$ $a = \frac{dv}{dt} = -cv^n$, $\int_{v_0}^{v} \frac{dv}{v^n} = -c\int_0^t dt$

$-ct = \frac{v^{1-n}}{1-n}\Big]_{v_0}^{v}$, $\underline{v = \left[v_0^{\,1-n} + c(n-1)t\right]^{\frac{1}{1-n}}}$

2/48 $a = k/x$, $v\,dv = \frac{k}{x}\,dx$

$\int_0^v v\,dv = k\int_{x_1}^{x} \frac{dx}{x}$ ; $\frac{v^2}{2} = k \ln \frac{x}{x_1}$

Thus $\frac{(600)^2}{2} = k \ln \frac{750}{7.5}$, $k = \frac{0.36}{2(4.605)} = 0.0391\ (km/s)^2$

at $x = 375$ mm, $a = \frac{0.0391}{375(10^{-6})} = \underline{104.2\ km/s^2}$

$\boxed{2/49}$ $a = v\dfrac{dv}{ds} = 3.22 - 0.004v^2$

$$\int_0^{v_B} \frac{v\,dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

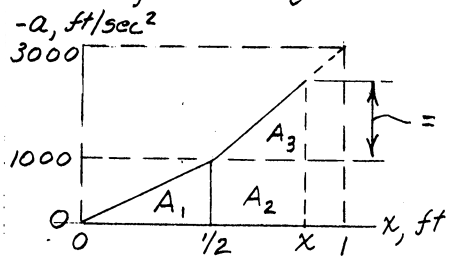
$$\frac{1}{2(-0.004)} \ln\left(3.22 - 0.004v^2\right)\Big|_0^{v_B} = 600$$

$$\ln\left[\frac{3.22 - 0.004v_B^2}{3.22}\right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$\underline{v_B = 28.3 \text{ ft/sec}}$

**2/50** $\int v\,dv = \int a\,dx$ or $\int_{40^2}^{0} dv^2 = -2\left(\begin{matrix}\text{area under}\\ a\text{-}x \text{ curve}\end{matrix}\right)$




$= \dfrac{x-\frac{1}{2}}{\frac{1}{2}}\,2000 = 4000(x-\tfrac{1}{2})$

$A_1 = \frac{1}{2}\,\frac{1}{2}(1000) = 250 \text{ ft}^2/\text{sec}^2$

$A_2 = (x-\tfrac{1}{2})1000$

$A_3 = \tfrac{1}{2}(x-\tfrac{1}{2})4000(x-\tfrac{1}{2})$

$A_1 + A_2 + A_3 = 2000x^2 - 1000x + 250$

so $-1600 = -2(2000x^2 - 1000x + 250)$

or $x^2 - 0.5x - 0.275 = 0$

$\underline{x = 0.831 \text{ ft}}$ $(x > 0.5 \text{ ft})$

2/51 Up:

$$a_u = -g - kv^2 = v\frac{dv}{dy}$$

$$\int_0^h dy = -\int_{v_0}^{0} \frac{v\,dv}{g+kv^2}$$

$$h = -\frac{1}{2k}\ln\left[g+kv^2\right]_{v_0}^{0} = \frac{1}{2k}\ln\left[\frac{g+kv_0^2}{g}\right]$$

$$h = \frac{1}{2(0.002)}\ln\left[\frac{32.2+0.002(100)^2}{32.2}\right] = \underline{120.8 \text{ ft}}$$

Down:

$$a_d = -g + kv^2 = v\frac{dv}{dy}$$

$$\int_h^0 dy = \int_0^{v_f}\frac{v\,dv}{-g+kv^2}$$

$$-h = \frac{1}{2k}\ln\left[-g+kv^2\right]\Big|_0^{v_f} = \frac{1}{2k}\ln\left[\frac{g-kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}\left(1-e^{-2kh}\right)}$$

$$= \sqrt{\frac{32.2}{0.002}\left(1-e^{-2(0.002)(120.8)}\right)} = \underline{78.5\ \frac{\text{ft}}{\text{sec}}}$$

**2/52** Up : $a_u = -g - kv^2 = \frac{dv}{dt}$

$$\int_0^{t_u} dt = -\int_{v_0}^{0} \frac{dv}{g+kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\frac{k}{g}}\right)$$

$$t_u = \frac{1}{\sqrt{32.2(0.002)}} \tan^{-1}\left(100\sqrt{\frac{0.002}{32.2}}\right) = \underline{2.63 \text{ sec}}$$

(Down) : $a_d = -g + kv^2 = \frac{dv}{dt}$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g+kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(v_f\sqrt{\frac{k}{g}}\right)$$

$$= \frac{1}{\sqrt{32.2(0.002)}} \tanh^{-1}\left(78.5\sqrt{\frac{0.002}{32.2}}\right)$$

$$= \underline{2.85 \text{ sec}}$$

(Refer to solution of Prob. 2/51)

2/53 For an acceleration of form $a = -g - kv^2$, we cite the results from Probs. 2/51 & 2/52

$$\begin{cases} t_u = \dfrac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\dfrac{k}{g}}\right) \\ h = \dfrac{1}{2k} \ln\left[\dfrac{g + kv_0^2}{g}\right] \end{cases}$$

For the numbers at hand:

$$t_u = \frac{1}{\sqrt{9.81\,(0.0005)}} \tan^{-1}\left(120\sqrt{\frac{0.0005}{9.81}}\right) = 10.11 \text{ s}$$

$$h = \frac{1}{2(0.0005)} \ln\left[\frac{9.81 + 0.0005\,(120)^2}{9.81}\right] = 550 \text{ m}$$

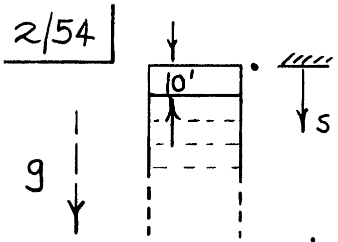
Down ($v$ = constant): $y = y_0 + v_{y_0} t$

$$0 = 550 - 4t_d$$

$$t_d = 137.6 \text{ s}$$

Flight time $t = t_u + t_d = 10.11 + 137.6 = \underline{147.7 \text{ s}}$

2/54




$s = \cancel{s_0'} + \cancel{v_0'}t + \frac{1}{2}gt^2$

When $s = 10$ ft,

$10 = \frac{1}{2}(32.2)t_{10'}^2$, $t_{10'} = 0.788$ sec

Time to pass first story from the top is $t_1 = t_{10'} - t_{0'} = 0.788 - 0 = \underline{0.788 \text{ sec}}$

$10^{th}$ story: $90 = \frac{1}{2}(32.2)t_{90'}^2$, $t_{90'} = 2.36$ sec

$100 = \frac{1}{2}(32.2)t_{100'}^2$, $t_{100'} = 2.49$ sec

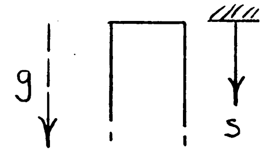
$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$

$100^{th}$ story: $990 = \frac{1}{2}(32.2)t_{990'}^2$

$1000 = \frac{1}{2}(32.2)t_{1000'}^2$

$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$

2/55




$$a = g - kv^2 = \frac{dv}{dt}$$

$$\int_0^t dt = \int_0^v \frac{dv}{g - kv^2}$$

(see Art. C/10): $t = \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v \Big|_0^v$

$$= \frac{1}{\sqrt{gk}} \tanh^{-1} \sqrt{\frac{k}{g}}\, v$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}\, t\right)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh\left(\sqrt{gk}\, t\right)$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk}\, t$$

or $\quad t = \dfrac{\cosh^{-1}\left(e^{sk}\right)}{\sqrt{gk}} = \dfrac{\cosh^{-1}\left(e^{0.005s}\right)}{0.401}$

| s, ft | t, sec |
|---|---|
| 0 | 0 |
| 10 | 0.795 |
| 90 | 2.54 |
| 100 | 2.70 |
| 990 | 14.06 |
| 1000 | 14.19 |

The time $t_1$ to pass first story is $t_1 = t_{10'} - t_0 = 0.795 - 0 = \underline{0.795 \text{ sec}}$

Similarly,

$\underline{t_{10} = 0.1592 \text{ sec}}$

$\underline{t_{100} = 0.1246 \text{ sec}}$

▶2/56 $\frac{dv}{dt} = ke^{-bt} - cv - g$, $\frac{dv}{dt} + cv = ke^{-bt} - g$

For the standard form for the solution of the first order linear differential equation,

$$e^{-\int c\,dt} = e^{-ct} \quad \&$$

$$v = Ae^{-ct} + e^{-ct}\int (ke^{-bt} - g)e^{ct}\,dt$$

$$= Ae^{-ct} + \frac{k}{c-b}e^{-bt} - \frac{g}{c}$$

when $t=0$, $v=0$ so $0 = A + \frac{k}{c-b} - \frac{g}{c}$, $A = \frac{g}{c} - \frac{k}{c-b}$

Thus $v = \left(\frac{g}{c} - \frac{k}{c-b}\right)e^{-ct} + \frac{k}{c-b}e^{-bt} - \frac{g}{c}$

or $\underline{v = \frac{g}{c}(e^{-ct} - 1) + \frac{k}{c-b}(e^{-bt} - e^{-ct})}$

▶2/57 $a = \frac{d^2x}{dt^2} = Kt - k^2x$, $\frac{d^2x}{dt^2} + k^2x = Kt$

$x = x_c + x_p = A \sin kt + B \cos kt + \frac{K}{k^2} t$

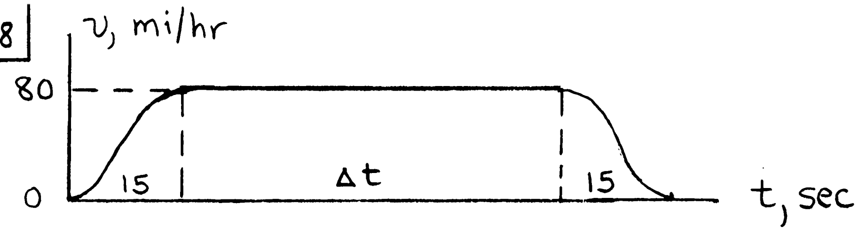
$\dot{x} = Ak \cos kt - Bk \sin kt + \frac{K}{k^2} = 0$ for $t = 0$

Thus $0 = Ak - 0 + \frac{K}{k^2}$, $A = -K/k^3$

$x = 0$ when $t = 0$, so $0 = 0 + B + 0$, $B = 0$

Thus $\underline{x = \frac{K}{k^3}(kt - \sin kt)}$

▶2/58




Cubic form : $v = a + bt + ct^2 + dt^3$

$v(0) = a = 0$, $\frac{dv}{dt} = b + 2ct + 3dt^2$

$\left.\frac{dv}{dt}\right|_{t=0} = 0 \Rightarrow b = 0$; $\left.\frac{dv}{dt}\right|_{t=15} = 30c + 675d = 0$

$v(15) = c(15)^2 + d(15)^3 = 80\left(\frac{5280}{3600}\right)$

Solve to obtain $c = 1.564 \text{ ft/sec}^3$, $d = -6.95(10^{-2})\frac{\text{ft}}{\text{sec}^4}$

$\therefore\ v = 1.564t^2 - 0.0695\,t^3$ for $0 \le t \le 15$ sec

$s = \int_0^t (1.564t^2 - 0.0695t^3)\,dt = 0.521t^3 - 0.01738\,t^4$

$s(15) = 880$ ft ; 880 ft also covered during decel.

Distance during cruise : $(2)(5280) - 2(880) = 8800$ ft

$\Delta s = v\Delta t$ : $8800 = 80\left(\frac{5280}{3600}\right)\Delta t$, $\Delta t = 75$ sec

$\therefore$ Run time $t = 75 + 2(15) = \underline{105 \text{ sec}}$

$a = \frac{dv}{dt} = 3.13t - 0.209t^2$

$\frac{da}{dt} = 3.13 - 0.417t = 0$, $t = 7.5$ sec

$a_{max} = 3.13(7.5) - 0.209(7.5)^2 = \underline{11.73 \text{ ft/sec}^2}$

$\boxed{2/59}$ $\underline{v}_{AV} = \dfrac{\Delta \underline{r}}{\Delta t} = \dfrac{0.03\underline{i} - 0.05\underline{j}}{0.02} = 1.5\underline{i} - 2.5\underline{j}\ \dfrac{m}{s}$

$v = |\underline{v}_{AV}| = \sqrt{1.5^2 + 2.5^2} = \underline{2.92\ m/s}$

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{-2.5}{1.5} = -\dfrac{5}{3}$, $\underline{\theta = -59.0^\circ}$

2/60

$$\underline{a}_{AV} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.3\underline{i} + 0.4\underline{j}}{0.1} = 3\underline{i} + 4\underline{j}\ \frac{m}{s^2}$$

$$a = |a_{AV}| = \sqrt{3^2 + 4^2} = \underline{5\ m/s^2}$$

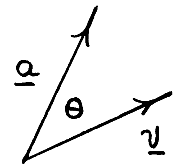
$$\tan\theta = \frac{a_y}{a_x} = \frac{4}{3}, \quad \underline{\theta = 53.1^\circ}$$

2/61

$$\underline{a}_{AV} = \frac{\Delta \underline{v}}{\Delta t}$$

$$4\underline{i} + 6\underline{j} = \left[\underline{v}_{3.67} - (6.12\underline{i} + 3.24\underline{j})\right] / 0.02$$

$$\underline{v} = \underline{v}_{3.67} = \underline{6.20\underline{i} + 3.36\underline{j} \text{ m/s}}$$



$$\cos\theta = \frac{\underline{v}\cdot\underline{a}}{va}$$

$$v = \sqrt{6.20^2 + 3.36^2} = 7.05 \text{ m/s}$$

$$a = \sqrt{4^2 + 6^2} = 7.21 \text{ m/s}^2$$

$$\cos\theta = \frac{(6.20\underline{i} + 3.36\underline{j})\cdot(4\underline{i} + 6\underline{j})}{(7.05)(7.21)} = \frac{45.0}{50.9} = 0.884$$

$$\underline{\theta = 27.9^\circ}$$

$\boxed{2/62}$

$$x = 2t^2 - 4t \qquad y = 3t^2 - \tfrac{1}{3}t^3$$

$$\dot{x} = 4t - 4 \qquad \dot{y} = 6t - t^2$$

$$\ddot{x} = 4 \text{ mm/s}^2 \qquad \ddot{y} = 6 - 2t$$

At $t = 2$ s: $\begin{cases} \dot{x} = 4(2) - 4 = 4 \text{ mm/s} \\ \dot{y} = 6(2) - 2^2 = 8 \text{ mm/s} \end{cases}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4^2 + 8^2} = \underline{8.94 \text{ mm/s}}$$

$$\theta_x = \tan^{-1}\frac{\dot{y}}{\dot{x}} = \tan^{-1}\frac{8}{4} = \underline{63.4^\circ}$$

At $t = 2$ s: $\begin{cases} \ddot{x} = 4 \text{ mm/s}^2 \\ \ddot{y} = 6 - 2(2) = 2 \text{ mm/s}^2 \end{cases}$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{4^2 + 2^2} = \underline{4.47 \text{ mm/s}^2}$$

$$\theta_x = \tan^{-1}\frac{\ddot{y}}{\ddot{x}} = \tan^{-1}\frac{2}{4} = \underline{26.6^\circ}$$

2/63 Use coordinate origin at A

$y$, $x$

Apply $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ at B:

$-4 = 0 + 0 - \frac{1}{2}(9.81)t_B^2$ , $t_B = 0.903$ s

Apply $x = x_0 + v_{x_0}t$ at B: $6 = 0 + v_0(0.903)$

$\underline{v_0 = 6.64 \text{ m/s}}$

Apply y-eq. @ C: $-8 = -\frac{1}{2}(9.81)t_c^2$

$t_c = 1.277$ s

Apply x-eq. @ C: $s + 6 = 6.64(1.277)$

$\underline{s = 2.49 \text{ m}}$

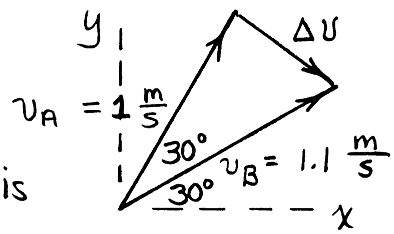
**2/64** $v = \dot{s} = \frac{t}{2}$ , $v_A = \frac{2}{2} = 1$ m/s, $v_B = \frac{2.2}{2} = 1.1 \frac{m}{s}$

$$\Delta v_x = v_{B_x} - v_{A_x} = 1.1 \cos 30° - 1.0 \cos 60° = 0.453 \ \frac{m}{s}$$

$$\Delta v_y = v_{B_y} - v_{A_y} = 1.1 \sin 30° - 1.0 \sin 60° = -0.316 \ \frac{m}{s}$$

$$\Delta v = \sqrt{0.453^2 + 0.316^2}$$

$$= 0.552 \ m/s$$




The average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{0.552}{0.20} = \underline{2.76 \ m/s^2}$$

$$\underline{a}_{av} = \frac{\Delta \underline{v}}{\Delta t} = \frac{0.453\underline{i} - 0.316\underline{j}}{0.20}$$

$$= \underline{2.26\underline{i} - 1.580\underline{j} \ \ m/s^2}$$

2/65 $a_x = 12t$, $\int_4^{v_x} dv_x = \int_0^t 12t\,dt$, $v_x = 4 + 6t^2$

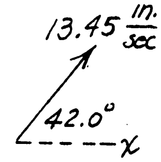
$y = 4t^3 - 3t$, $v_y = \dot{y} = 12t^2 - 3$, $a_y = \ddot{y} = 24t$

when $t = 1$ sec, $v_x = 4 + 6(1^2) = 10$ in./sec

$v_y = 12(1^2) - 3 = 9$ in./sec

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 9^2} = \underline{13.45 \text{ in./sec}}$

$\theta_x = \tan^{-1} v_y/v_x = \tan^{-1} 9/10 = \underline{42.0°}$

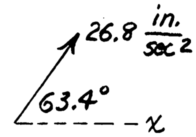



$a_x = 12(1) = 12$ in./sec$^2$

$a_y = 24(1) = 24$ in./sec$^2$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{12^2 + 24^2} = \underline{26.8 \frac{\text{in.}}{\text{sec}^2}}$

$\theta_x = \tan^{-1} a_y/a_x = \tan^{-1} 24/12 = \underline{63.4°}$



2/66

$$\underline{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\underline{i} + \left(\frac{t^4}{12}\right)\underline{j}$$

$$\underline{v} = \dot{\underline{r}} = \left(2t^2 - 3t\right)\underline{i} + \left(\frac{1}{3}t^3\right)\underline{j}$$

$$\underline{a} = \dot{\underline{v}} = \left(4t - 3\right)\underline{i} + \left(t^2\right)\underline{j}$$

At $t = 2s$

$$\begin{cases} \underline{v} = (2\cdot 2^2 - 3\cdot 2)\underline{i} + \frac{1}{3}2^3\underline{j} = 2\underline{i} + \frac{8}{3}\underline{j} \ \ \text{mm/s} \\ \underline{a} = (4\cdot 2 - 3)\underline{i} + 2^2\underline{j} = 5\underline{i} + 4\underline{j} \ \ \text{mm/s}^2 \end{cases}$$

$$\cos\theta = \frac{\underline{v}\cdot\underline{a}}{va} = \frac{\left(2\underline{i} + \frac{8}{3}\underline{j}\right)\cdot\left(5\underline{i} + 4\underline{j}\right)}{\sqrt{2^2 + \left(\frac{8}{3}\right)^2}\sqrt{5^2 + 4^2}}$$

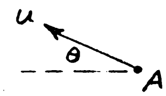
$$\underline{\theta = 14.47^\circ}$$

At $t = 3s$

$$\begin{cases} \underline{v} = (2\cdot 3^2 - 3\cdot 3)\underline{i} + \left(\frac{1}{3}3^3\right)\underline{j} = 9\underline{i} + 9\underline{j} \ \ \frac{\text{mm}}{\text{s}} \\ \underline{a} = (4\cdot 3 - 3)\underline{i} + (3^2)\underline{j} = 9\underline{i} + 9\underline{j} \ \ \frac{\text{mm}}{\text{s}^2} \end{cases}$$

$$\underline{v} \parallel \underline{a} \Rightarrow \underline{\theta = 0}$$

2/67 | From Sample Prob. 2/6



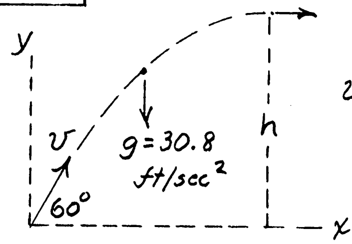

$$2s = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g}$$

But $2s = 22$ ft, $u\cos\theta = 30$ ft/sec, $u\sin\theta = v_y$

so $v_y = \frac{2sg}{2u\cos\theta} = \frac{22(32.2)}{2(30)} = \underline{11.81 \text{ ft/sec}}$

Also, $h = \frac{u^2\sin^2\theta}{2g} = v_y^2/2g = \frac{(11.81)^2}{2(32.2)} = \underline{2.16 \text{ ft}}$

2/68 | $v = 600 \text{ mi/hr} \equiv 880 \text{ ft/sec}$




$a_y = -30.8 \text{ ft/sec}^2$

$v_y = v_{y_0} + at\,;$

$0 = 880 \sin 60° - 30.8\, t$

$t = \dfrac{880(0.866)}{30.8} = \underline{24.7 \text{ sec}}$

$v_y^2 = v_{y_0}^2 + 2ay\,;\quad 0 = (880 \sin 60°)^2 - 2(30.8)h$

$h = \dfrac{[880(0.866)]^2}{2(30.8)} = 9429 \text{ ft}$

or $\quad h = \dfrac{9429}{5280} = \underline{1.786 \text{ mi}}$

2/69 Set up $x$-$y$ axes at the initial location of G.

$$\left.\begin{array}{ll} x = x_0 + v_{x_0} t & : \quad 3 = (v_0 \cos\theta) t \\ y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 & : \quad 3.5 = (v_0 \sin\theta) t - 16.1 t^2 \\ v_y = v_{y_0} - g t & : \quad 0 = v_0 \sin\theta - 32.2 t \end{array}\right\}$$

Solve simultaneously : $\left\{\begin{array}{l} t = 0.466 \text{ sec} \\ \underline{v_0 = 16.33 \text{ ft/sec}} \\ \underline{\theta = 66.8^\circ} \end{array}\right.$

2/70 $v_y = \dot{y} = 8t$, $y = 4t^2 + C_1$; $y = 2$ ft when $t = 0$

so $C_1 = 2$ ft & $y = 4t^2 + 2$ ft

$a_x = \ddot{x} = 4t$, $\dot{x} = 2t^2 + C_2$; $\dot{x} = 0$ when $t = 0$ so $C_2 = 0$

$x = \frac{2t^3}{3} + C_3$; $x = 0$ when $t = 0$ so $C_3 = 0$

Eliminate $t$ & get $y = 4\sqrt[3]{\frac{9x^2}{4}} + 2$

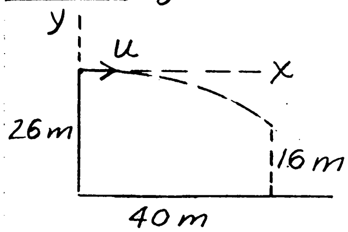
or $\underline{(y-2)^3 = 144x^2}$

When $x = 18$ ft, $\frac{2t^3}{3} = 18$, $t^3 = 27$, $t = 3$ sec

& $\dot{x} = 2(3)^2 = 18 \frac{ft}{sec}$, $\dot{y} = 8(3) = 24 \frac{ft}{sec}$

$v = \sqrt{\overline{18}^2 + \overline{24}^2} = \underline{30 \text{ ft/sec}}$

**2/71** $a_y = -g$ so $y = 0 - \frac{1}{2}gt^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$

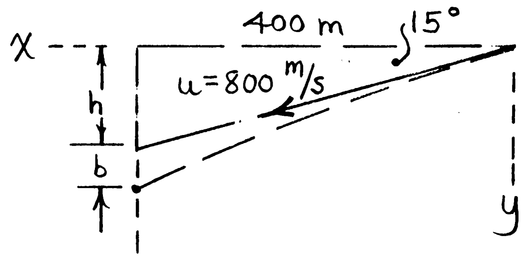



$= 1.428$ s

$x = ut$; $u = 40/1.428$

$= \underline{28.0 \text{ m/s}}$

2/72




$h = 400 \tan 15°$

$= 107.2 \text{ m}$

The equation of the path (Sample Problem 2/6)

is $\quad y = x \tan\theta + \dfrac{g x^2}{2u^2} \sec^2\theta \quad$ (+ sign here)

So $\quad b = \dfrac{g x^2}{2u^2} \sec^2\theta$

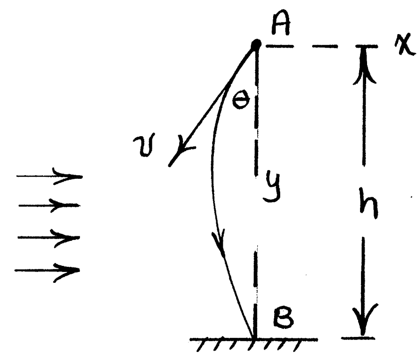
At $x = 400$ m: $b = \dfrac{9.81(400)^2}{2(800)^2} \sec^2 15°$

$= \underline{1.314 \text{ m}}$

2/73

$a_x = a$

$a_y = g$



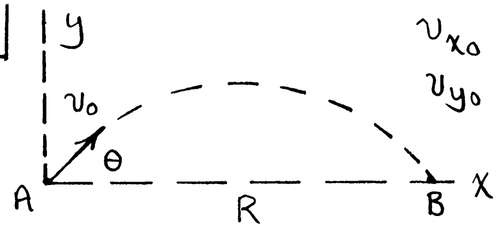

$x = v_{x_0} t + \frac{1}{2} a_x t^2 \ : \ x = -v\sin\theta\, t + \frac{1}{2} a t^2$

$y = v_{y_0} t + \frac{1}{2} g t^2 \ : \ y = v\cos\theta\, t + \frac{1}{2} g t^2$

At B: $\begin{cases} x = 0 \quad \text{so} \quad 0 = t\left(-v\sin\theta + \frac{1}{2} a t\right), \ t = \dfrac{2v\sin\theta}{a} \\ y = h \quad \text{so} \quad h = v\cos\theta\left(\dfrac{2v\sin\theta}{a}\right) + \dfrac{1}{2} g \left(\dfrac{2v\sin\theta}{a}\right)^2 \end{cases}$

$$\underline{h = \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)}$$

2/74




$v_{x_0} = v_0 \cos\theta$

$v_{y_0} = v_0 \sin\theta$

$x = x_0 + v_{x_0} t$ @ B: $R = 0 + (v_0 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $0 = 0 + (v_0 \sin\theta) t_f - \frac{g}{2} t_f^2$ (2)

(2): $t_f = 0, \; \frac{2 v_0 \sin\theta}{g}$ ($t = 0$ is launch time)

(1): $R = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right) = \frac{v_0^2 \sin 2\theta}{g}$

$\frac{dR}{d\theta} = 0$: $\frac{v_0^2}{g} 2 \cos 2\theta = 0 \Rightarrow \underline{\theta = 45°}$

$R_{max} = \frac{v_0^2 \sin(2 \cdot 45°)}{g} = \underline{\frac{v_0^2}{g}}$

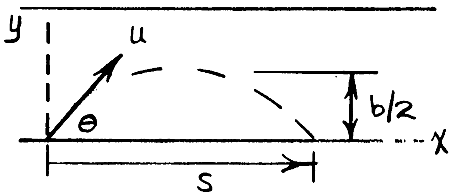
2/75 | Angle $\theta$ for maximum range and, hence, minimum $u$ is $45°$

From Sample Prob. 2/6 range is

$$2s = \frac{u^2}{g}\sin 2\theta, \quad \text{so} \quad 12(10^3) = \frac{u^2}{9.81}(1)$$

$$u = \sqrt{9.81(12000)} = \underline{343 \text{ m/s}}$$

2/76




$a_y = -\frac{eE}{m}$, constant

$a_x = 0$

$v_y^2 - v_{y_0}^2 = 2ay$ : At top, $0 - (u\sin\theta)^2 = 2\left(-\frac{eE}{m}\right)\frac{b}{2}$

$$\underline{E = \frac{mu^2\sin^2\theta}{eb}}$$

$v_y = v_{y_0} + a_y t$ : At top, $0 = u\sin\theta - \frac{eE}{m}t$

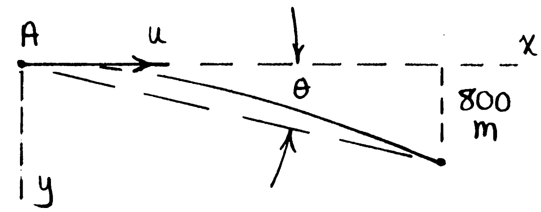
$$t = \frac{mu\sin\theta}{eE}$$

$x = v_{x_0}t$ : $s = (u\cos\theta)(2t) = u\cos\theta\left(\frac{2mu\sin\theta}{eE}\right)$

$$= u\cos\theta\left(\frac{2mu\sin\theta}{e\,\frac{mu^2\sin^2\theta}{eb}}\right) = \underline{2b\cot\theta}$$

2/77

$u = \frac{1000}{3.6} = 278\ \frac{m}{s}$




y-dir. : $y = v_{y_0}t + \frac{1}{2}gt^2$

$800 = 0 + \frac{1}{2}(9.81)t^2, \quad t = 12.77\ s$

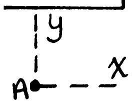
x-dir. : $x = v_{x_0}t + \frac{1}{2}a_x t^2$

$= 278(12.77) + \frac{1}{2}\left(\frac{9.81}{2}\right)(12.77)^2$

$= 3950\ m$

$\theta = \tan^{-1}\frac{800}{3950} = \underline{11.46°}$

2/78




$v_{x_0} = v_0 \cos\theta = 45 \cos 40° = 34.5$ ft/sec

$v_{y_0} = v_0 \sin\theta = 45 \sin 40° = 28.9$ ft/sec

$x = x_0 + v_{x_0} t$ at wall: $60 = 0 + 34.5\,t$

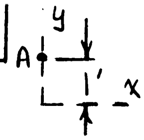
$t = 1.741$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at wall:

$y = 1 + 28.9(1.741) - 16.1(1.741)^2 = 2.57$ ft

So water strikes wall 2.57 ft above B.

2/79




$v_{x_0} = 45\cos\theta$, $v_{y_0} = 45\sin\theta$

Strategy: Determine $\theta$ for hitting top point of wall; take larger of two values.

$x = x_0 + v_{x_0}t$ : $60 = 0 + (45\cos\theta)t$, $t = \dfrac{60}{45\cos\theta}$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$: $3 = 1 + 45\sin\theta\left(\dfrac{60}{45\cos\theta}\right) - 16.1\left(\dfrac{60}{45\cos\theta}\right)^2$

Use $\dfrac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta$ & rearrange:

$$28.6\tan^2\theta - 60\tan\theta + 30.6 = 0$$

$$\Rightarrow \begin{cases} \tan\theta = 0.879, & \theta = 41.3° \\ \tan\theta = 1.218, & \theta = 50.6° \end{cases}$$

$\therefore$ $\theta$ must be just larger than $\underline{50.6°}$

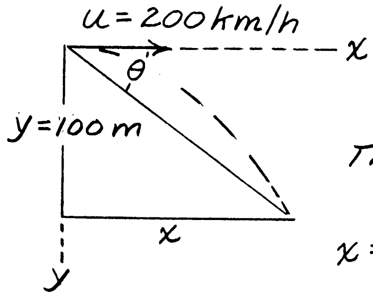
$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ at impact: $0 = 1 + 45\sin 50.6°\,t - 16.1t^2 \Rightarrow t = 2.19$ sec

$x = x_0 + v_{x_0}t$: $R = (45\cos 50.6°)\,2.19 = 62.5$ ft

$\therefore$ <u>Water lands 2.50 ft to right of B.</u>

2/80 For y-motion $a_y = g = 9.81\ m/s^2$

$y = \frac{1}{2} g t^2$




For x-motion $a_x = 0$ so

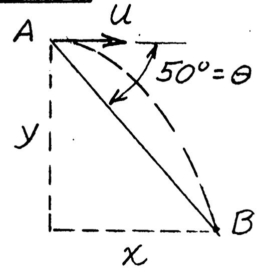
$x = ut$

Thus $y = \frac{g x^2}{2u^2}$, $x = u\sqrt{\frac{2y}{g}}$

$$x = \frac{200}{3.6}\sqrt{\frac{2(100)}{9.81}} = 251\ m$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{100}{251} = \underline{21.7°}$$

**2/81**



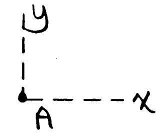

$$y = \frac{1}{2} g t^2 , \quad x = ut$$

$$\tan\theta = \frac{y}{x} = \frac{gt}{2u}$$

$$\text{so } u = \frac{gt}{2\tan\theta} = \frac{9.81\,(3.5)}{2(1.1918)}$$

$$= \underline{14.41 \text{ m/s}}$$

2/82 (a) $v_0 = 140$ ft/sec and $\theta = 8°$:




$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (140 \cos 8°)t$

$$\underline{t = 1.443 \text{ sec}}$$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

$-(7.5 - h) = 0 + 140 \sin 8° (1.443) - \frac{1}{2}(32.2)(1.443)^2$

$$\underline{h = 2.10 \text{ ft}}$$

(b) $v_0 = 120$ ft/sec and $\theta = 12°$:

$x = x_0 + v_{x_0} t$ @ B: $200 = 0 + (120 \cos 12°)t$

$$\underline{t = 1.704 \text{ sec}}$$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B:

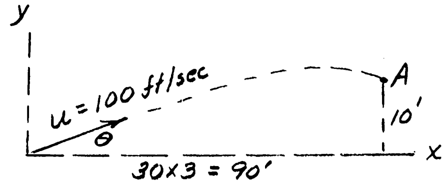
$-(7.5 - h) = 0 + (120 \sin 12°)(1.704) - \frac{1}{2}(32.2)(1.704)^2$

$$\underline{h = 3.27 \text{ ft}}$$

(In baseball, the time of flight is critical; low trajectories, even with one hop, are better.)

2/83

$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$




Let $m = \tan\theta$

$\sec^2\theta = 1 + \tan^2\theta = 1 + m^2$

$$y = xm - \frac{gx^2}{2u^2}(1+m^2), \quad m^2 - \frac{2u^2}{gx}m + \left(\frac{2u^2y}{gx^2} + 1\right) = 0$$

At A, $m^2 - \frac{2(10^2)^2}{32.2(90)}m + \left(\frac{2(10^2)^2 10}{32.2(90)^2} + 1\right) = 0$

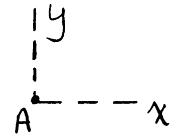
$$m^2 - 6.901m + 1.7668 = 0$$

$$m = \frac{6.901}{2} \pm \frac{1}{2}\sqrt{(6.901)^2 - 4(1.7668)}$$

$$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266 \text{ or } 6.635$$

$$\theta = \tan^{-1}m = \underline{14.91°} \qquad (\text{or } 81.4°)$$

**2/84** Set up x-y coordinates with origin at A.




$$v_{x_0} = 25 \cos\theta$$

$$v_{y_0} = 25 \sin\theta$$

Evaluate $v_y^2 = v_{y_0}^2 - 2g(y - y_0)$ at ceiling:

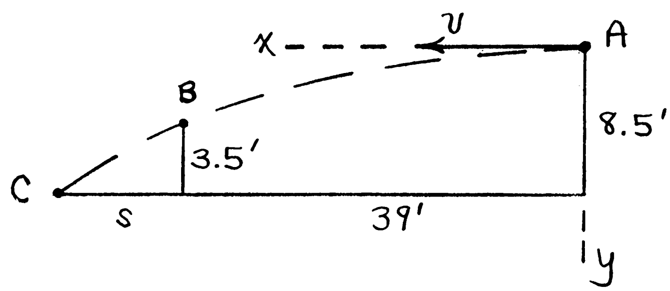
$$0^2 = (25 \sin\theta)^2 - 2(9.81)(5-0)$$

$$\theta = 23.3°$$

Range $R = \dfrac{v_0^2 \sin 2\theta}{g} = \dfrac{25^2 \sin(2 \cdot 23.3°)}{9.81}$

$$\underline{R = 46.4 \text{ m}}$$

2/85




$a_x = 0$ : $x = v_{x_0} t$ , $39 = v t_B$

$a_y = g$ : $y = v_{y_0} t + \frac{1}{2} g t^2$

At B : $8.5 - 3.5 = 0 + \frac{1}{2} 32.2 t_B^2$, $t_B = 0.557$ sec

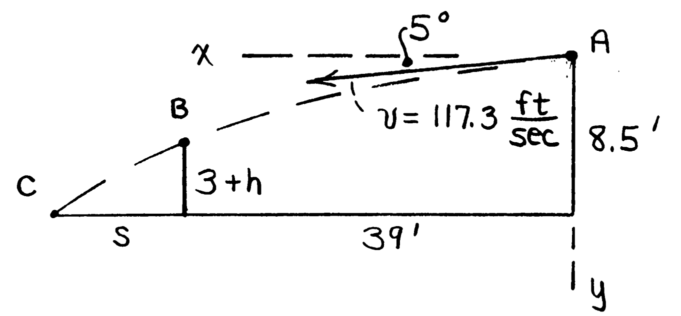
Then $v = \frac{39}{t_B} = \frac{39}{0.557} = 70.0$ ft/sec

$\underline{(47.7 \text{ mi/hr})}$

At C : $8.5 = \frac{1}{2}(32.2) t_C^2$, $t_C = 0.727$ sec

$s + 39 = 70.0(0.727)$ , $\underline{s = 11.85 \text{ ft}}$

2/86




$a_x = 0,\quad x = v_{x_0} t\ :\ 39 = 117.3 \cos 5°\ t_B,\ t_B = 0.334\text{ sec}$

$a_y = g,\quad y = v_{y_0} t + \frac{1}{2} g t^2 :\quad$ At B,

$$5.5 - h = 117.3 \sin 5° (0.334) + \frac{1}{2}(32.2)(0.334)^2$$

$$h = \underline{0.296\text{ ft}} \quad \text{or} \quad h = \underline{3.55\text{ in.}}$$

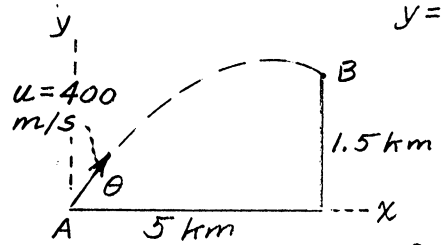
At C: $\quad 8.5 = 117.3 \sin 5°\ t_C + 16.1\, t_C^{\,2}$

$$t_C = 0.475\text{ sec}$$

x-equation at C: $\quad 39 + s = 117.3 \cos 5° (0.475)$

$$\underline{s = 16.57\text{ ft}}$$

**2/87** Eq. of trajectory (Sample Problem 2/6)




$$y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$$

$$= x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$$

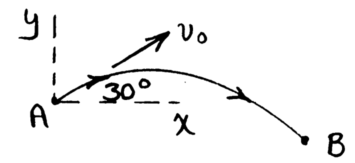
Substitute values & get

$$1500 = 5000\tan\theta - \frac{9.81(5000)^2}{2(400)^2}(1+\tan^2\theta)$$

or $\tan^2\theta - 6.524\tan\theta + 2.957 = 0$

solution gives roots $\underline{\theta_1 = 26.1^\circ}$ & $\underline{\theta_2 = 80.6^\circ}$

2/88 Set up $x$-$y$ axes at A, target at B:



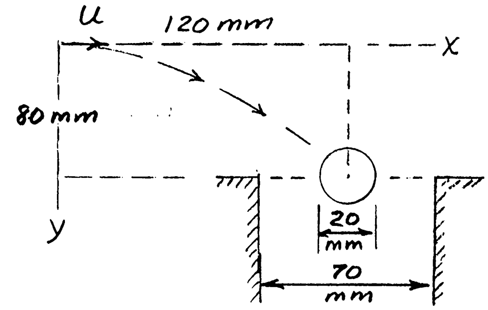

$$\left.\begin{aligned} x\text{-eq.}: \quad x_B &= (v_0 \cos 30°)t \\ y\text{-eq.}: \quad y_B &= (v_0 \sin 30°)t - \tfrac{1}{2}gt^2 \end{aligned}\right\}$$

For $x_B = 12'$, $y_B = -0.333'$: $\begin{cases} v_0 = 20.6 \text{ ft/sec} \\ t = 0.672 \text{ sec} \end{cases}$

For $x_B = 14'$, $y_B = -0.333'$: $\begin{cases} v_0 = 22.4 \dfrac{\text{ft}}{\text{sec}} \\ t = 0.723 \text{ sec} \end{cases}$

So the range is $\underline{20.6 \leq v_0 \leq 22.4 \text{ ft/sec}}$

2/89




$x = ut, \quad y = \frac{1}{2}gt^2$

$x = u\sqrt{\frac{2y}{g}}, \quad u = x\sqrt{\frac{g}{2y}}$

$x_{max} = 120 + \frac{70}{2} - \frac{20}{2} = 145 \text{ mm}$

$x_{min} = 120 - \frac{70}{2} + \frac{20}{2} = 95 \text{ mm}$

$u_{max} = 0.145\sqrt{\frac{9.81}{2(0.080)}} = \underline{1.135 \text{ m/s}}$

$u_{min} = 0.095\sqrt{\frac{9.81}{2(0.080)}} = \underline{0.744 \text{ m/s}}$

2/90 Set up x-y coordinates at A

x-eq.: $x_B = (36 \cos\theta)t$

y-eq.: $y_B = (36 \sin\theta)t - 16.1t^2$

Solutions:

For $x_B = 40'$, $y_B = -\frac{22}{12}'$ (top of stake):

$\theta = 34.3°$ or $\theta = 53.1°$

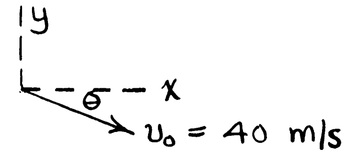
For $x_B = 40'$, $y_B = -3'$ (bottom of stake):

$\theta = 31.0°$ or $\theta = 54.7°$

Ranges: $31.0° \leq \theta \leq 34.3°$

or $\underline{53.1° \leq \theta \leq 54.7°}$

2/91 Set up $x$-$y$ coordinates with origin at release point :




$x = x_0 + v_{x_0} t$ at mitt : $20 = (40 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at mitt :

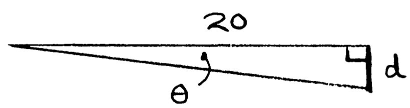
$$-1.8 = 0 + (-40 \sin\theta) t_f - \frac{9.81}{2} t_f^{\,2} \qquad (2)$$

(1) : $t_f = \dfrac{1}{2\cos\theta}$

(2) : $-1.8 = -40 \sin\theta \left(\dfrac{1}{2\cos\theta}\right) - \dfrac{9.81}{2}\left(\dfrac{1}{2\cos\theta}\right)^2$

Use $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$ : $1.226 \tan^2\theta + 20 \tan\theta - 0.574 = 0$

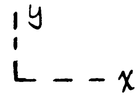
$$\Rightarrow \quad \theta = 1.640^\circ$$




$d = 20 \tan 1.640$
$= 0.573'$

$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$

2/92 | Use $x$-$y$ coordinates with origin at the release point :




$x = x_0 + v_{x_0} t$ @ hoop : $13.75 = 0 + (v_0 \cos 50°) t_f$

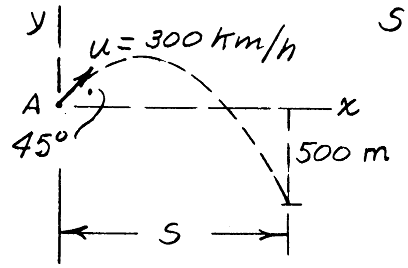
$t_f = 21.4/v_0$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ hoop :

$3 = 0 + v_0 \sin 50° \left( \frac{21.4}{v_0} \right) - 16.1 \left( \frac{21.4}{v_0} \right)^2$

$\underline{v_0 = 23.5 \text{ ft/sec}}$

2/93 $u = 300/3.6 = 83.33 \text{ m/s}$




$s = ut\cos\theta$

$= 83.33(0.7071)t = 58.93\,t$

$y = ut\sin\theta + \frac{1}{2}a_y t^2$

$-500 = 83.33\,t(0.7071) + \frac{1}{2}(-9.81)t^2$

or $t^2 - 12.01\,t - 101.94 = 0$

$$t = \frac{12.01 \pm \sqrt{\overline{12.01}^2 + 4(101.94)}}{2} = \underline{17.75 \text{ s}}$$

(or $-5.74$ s)

$s = 58.93(17.75) = 1046.2 \text{ m}$ or $\underline{s = 1.046 \text{ km}}$

**2/94**

$$\underline{a} = -k\underline{v} - g\underline{j}$$

$$a_x \underline{i} + a_y \underline{j} = -k(v_x \underline{i} + v_y \underline{j}) - g\underline{j}$$

$$\therefore \; a_x = -kv_x, \quad a_y = -kv_y - g$$

$\underline{x}$:

$$a_x = \frac{dv_x}{dt} = -kv_x$$

$$\int_{v_{x_0}}^{v_x} \frac{dv_x}{v_x} = -\int_0^t k\,dt \Rightarrow v_x = v_{x_0} e^{-kt}$$

$$\text{or } \underline{v_x = (v_0 \cos\theta) e^{-kt}}$$

$$v_x = \frac{dx}{dt} = v_{x_0} e^{-kt}$$

$$\int_0^x dx = \int_0^t v_{x_0} e^{-kt} dt$$

$$x = \frac{v_{x_0}}{k}\left[1 - e^{-kt}\right] = \underline{\frac{v_0 \cos\theta}{k}\left[1 - e^{-kt}\right]}$$

$\underline{y}$:

$$a_y = \frac{dv_y}{dt} = -kv_y - g$$

$$\int_{v_{y_0}}^{v_y} \frac{dv_y}{kv_y + g} = -\int_0^t dt$$

$$\Rightarrow v_y = \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k} = \underline{\left[v_0 \sin\theta + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}$$

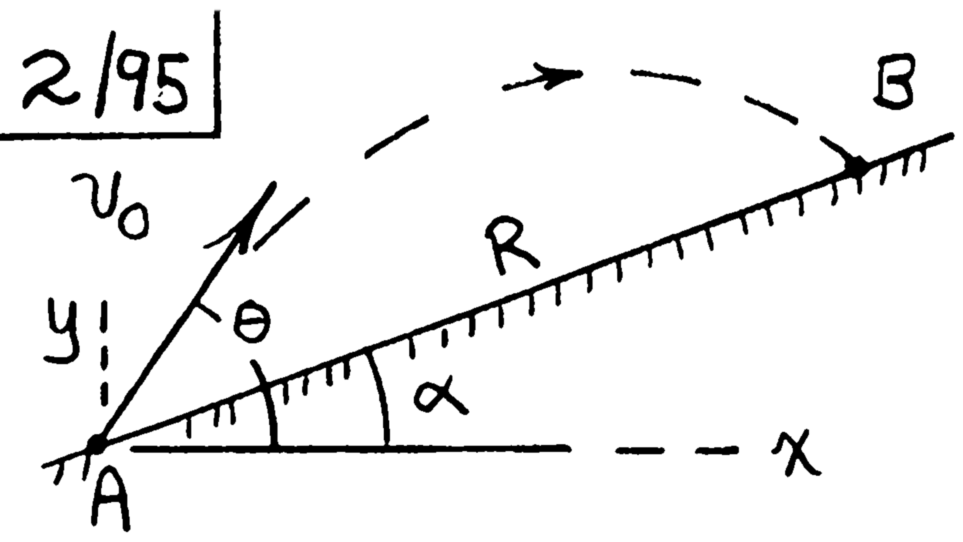
$$\int_0^y dy = \int_0^t \left\{\left[v_{y_0} + \frac{g}{k}\right] e^{-kt} - \frac{g}{k}\right\} dt$$

$$\underline{y = \frac{1}{k}\left[v_0 \sin\theta + \frac{g}{k}\right]\left[1 - e^{-kt}\right] - \frac{g}{k} t}$$

Terminal velocity $(t \to \infty)$: $v_x \to 0$

$$\underline{v_y \to -\frac{g}{k}}$$

▶ 2/95




$x = x_0 + v_{x_0} t$ @ B: $\quad R\cos\alpha = (v_0 \cos\theta) t_f$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R \sin\alpha = (v_0 \sin\theta) t_f - \frac{1}{2} g t_f^2$

x-eq: $\quad t_f = \dfrac{R\cos\alpha}{v_0 \cos\theta}$

y-eq: $\quad R\sin\alpha = (v_0 \sin\theta)\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) - \frac{1}{2} g \left(\dfrac{R\cos\alpha}{v_0 \cos\theta}\right)^2$

$$\Rightarrow \quad R = \frac{2 v_0^2 \cos^2\theta}{g\cos\alpha}\left(\tan\theta - \tan\alpha\right)$$

$$\frac{dR}{d\theta} = 0: \quad \frac{4 v_0^2 \cos\theta(-\sin\theta)}{g\cos\alpha}\left(\tan\theta - \tan\alpha\right) + \frac{2 v_0^2 \cos^2\theta}{g \cos\alpha}\frac{1}{\cos^2\theta} = 0$$

$$\frac{2 v_0^2}{g\cos\alpha}\left[2\cos\theta\sin\theta\left(\tan\alpha - \tan\theta\right) + 1\right] = 0$$

$$\Rightarrow \quad 2\cos\theta \sin\theta\left(\tan\alpha - \frac{\sin\theta}{\cos\theta}\right) + 1 = 0$$

$$(2\cos\theta\sin\theta)\tan\alpha - 2\sin^2\theta + 1 = 0$$

$$\sin 2\theta \tan\alpha - 2\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta\right) + 1 = 0$$

$$\sin 2\theta \tan\alpha + \cos 2\theta = 0$$

$$\tan 2\theta = -\frac{1}{\tan\alpha}$$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$

$$= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\therefore \quad \theta = \underline{\frac{90^\circ + \alpha}{2}}$$

Specific results: $\left\{ \begin{array}{ll} \alpha = 0, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \alpha = 45^\circ, & \theta = 67.5^\circ \end{array} \right.$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180^\circ - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$

$$= 180^\circ - (90^\circ - \alpha) = 90^\circ + \alpha$$

$$\therefore \quad \theta = \underline{\frac{90^\circ + \alpha}{2}}$$

Specific results :
$$\begin{cases} \alpha = 0, & \theta = 45^\circ \\ \alpha = 30^\circ, & \theta = 60^\circ \\ \alpha = 45^\circ, & \theta = 67.5^\circ \end{cases}$$

▸2/96 Eq. of path: $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

Let $m = \tan\theta$, $\dfrac{1}{\cos^2\theta} = \sec^2\theta = 1 + \tan^2\theta = 1 + m^2$

Thus $y = mx - \dfrac{gx^2}{2u^2}(1+m^2)$

or $m^2 - \left(\dfrac{2u^2}{gx}\right)m + \left(1 + \dfrac{2u^2 y}{gx^2}\right) = 0$

Roots are equal if discriminant $= 0$

Thus $\left(\dfrac{2u^2}{gx}\right)^2 - 4\left(1 + \dfrac{2u^2 y}{gx^2}\right) = 0$

Solve for y & get $y = \dfrac{u^2}{2g} - \dfrac{gx^2}{2u^2}$

a vertical parabola

$\boxed{2/97}$ $a_n = v^2/\rho = (0.6)^2/0.4 = 0.9\ m/s^2$

(a) $a_t = \dot{v} = 0$, so $\underline{a = a_n = 0.9\ m/s^2}$

(b) $a_t = \dot{v} = 1.2\ m/s^2$, so $a = \sqrt{a_t^2 + a_n^2}$

$= \sqrt{1.2^2 + 0.9^2} = \underline{1.5\ m/s^2}$

2/98 $a_1$: speed is increasing, no path curvature.

$a_2$: speed increasing, car turning to left.

$a_3$: speed stationary, car turning to left.

$a_4$: Speed decreasing, car turning to left.

$a_5$: Speed decreasing, no path curvature.

$a_6$: Speed decreasing, car turning to right.

2/99 $a_n = v^2/\rho = \left(\frac{45}{30}44\right)^2/800 = 5.45$ ft/sec$^2$

$a = \sqrt{a_n^2 + a_t^2}$; $a_t = \sqrt{a^2 - a_n^2} = \sqrt{10^2 - (5.45)^2} = \underline{8.39 \text{ ft/sec}^2}$

or $a_t = \frac{8.39}{44} 30 = \underline{5.72 \text{ mi/hr each second}}$ increasing or decreasing speed

2/100 $a = a_n = v^2/\rho$, $v = \sqrt{\rho a_n} = \sqrt{(100 - 0.6)0.5(9.81)}$

$= 22.08$ m/s

or $v = 22.08(3.6) = \underline{79.5 \text{ km/h}}$

$\underline{2/101}$ $a = a_n = v^2/\rho$, $v^2 = \rho a_n$

$\rho = 98 + 2 = 100$ m, $v^2 = 100(0.4)(9.81)$

$$\underline{v = 19.81 \ \frac{m}{s} \ \text{or} \ 71.3 \ \frac{km}{h}}$$

$\underline{2/102}$ $a = a_n = v\dot{\beta} = \frac{20(1.852)}{3.6}\frac{\pi}{2}\frac{1}{60} = \underline{0.269\ m/s^2}$

$\underline{2/103}$ $v = v_0 + a_t t$ : $\frac{50}{3.6} = \frac{100}{3.6} + 12 a_t$

$$a_t = -1.157 \text{ m/s}^2$$

$a = \sqrt{a_t^2 + a_n^2}$ : $2 = \sqrt{1.157^2 + a_n^2}$

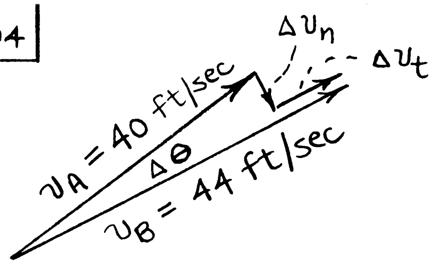
$$a_n = 1.631 \text{ m/s}^2$$

$v_6 = v_0 + a_t t = \frac{100}{3.6} - 1.157(6) = 20.8 \text{ m/s}$

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{20.8^2}{1.631}$

$$= \underline{266 \text{ m}}$$

2/104

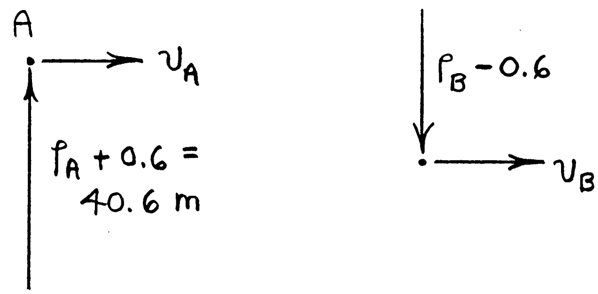



$$\Delta\theta = \frac{36-26}{180}\pi$$
$$= 0.1745 \text{ rad}$$
$$v_{AV} = 42 \text{ ft/sec}$$

$$a_n = \frac{\Delta v_n}{\Delta t} = \frac{v\Delta\theta}{\Delta t} = \frac{42(0.1745)}{3.84-3.64} = \underline{36.7\ \frac{\text{ft}}{\text{sec}^2}}$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{44-40}{0.2} = \underline{20 \text{ ft/sec}^2}$$

2/105




$$v_A = \frac{50}{3.6} = 13.89 \text{ m/s}, \quad v_B = \frac{100}{3.6} = 27.8 \text{ m/s}$$

$$v_B = v_A + a_t t, \quad a_t = \frac{27.8 - 13.89}{10} = 1.389 \text{ m/s}^2$$

At A: $a_n = \dfrac{v^2}{\rho} = \dfrac{13.89^2}{40.6} = 4.75 \text{ m/s}^2$

Total: $a_A = \sqrt{4.75^2 + 1.389^2} = 4.95 \text{ m/s}^2$

At B: For $a_A = a_B = 4.95 \text{ m/s}^2$,

$$4.95 = \sqrt{1.389^2 + a_n^2}$$

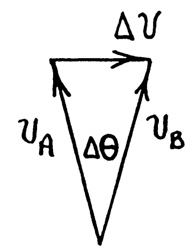
$$a_n = 4.75 = \frac{27.8^2}{\rho_B - 0.6}, \quad \underline{\rho_B = 163.0 \text{ m}}$$

2/106 $v_A = v_B = v = 2$ m/s

$\Delta v = 2v \sin\frac{\Delta\theta}{2} = 4 \sin\frac{\Delta\theta}{2}$ m/s

$\Delta t = \frac{r\Delta\theta}{v} = \frac{0.8\,\Delta\theta}{2} = 0.4\Delta\theta$ s

$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4\sin\frac{\Delta\theta}{2}}{0.4\,\Delta\theta} = 5\,\frac{\sin\frac{\Delta\theta}{2}}{\Delta\theta/2}$




| | $\Delta\theta^\circ$ | $\frac{\Delta\theta}{2}^\circ$ | $\frac{\Delta\theta}{2}$ rad | $\sin\frac{\Delta\theta}{2}$ | $a_{av}$, m/s$^2$ | % diff. |
|---|---|---|---|---|---|---|
| (a) | 30° | 15° | 0.262 | 0.259 | 4.94 | 1.1 |
| (b) | 15° | 7.5° | 0.1309 | 0.1305 | 4.99 | 0.3 |
| (c) | 5° | 2.5° | 0.0436 | 0.0436 | 4.998 | 0.03 |

$a_n = \frac{v^2}{\rho} = \frac{2^2}{0.8} = \underline{5 \text{ m/s}^2}$

**2/107** $a_n = v^2/r$, where $a_n = g = g_0 \frac{R^2}{r^2}$

So $v^2 = gr = g_0 \frac{R^2}{r} = g_0 \frac{R^2}{R+h}$

$$= 9.821 \frac{[6.371(10^6)]^2}{[6.371(10^6) + 0.320(10^6)]}$$

$$v = 7.72(10^3)\ \text{m/s} \quad \text{or} \quad \underline{27.8(10^3)\ \frac{\text{km}}{\text{h}}}$$

2/108 From $a_n = \frac{v^2}{\rho}$, $v = \sqrt{a_n \rho} = \sqrt{0.8 g \rho}$

$v_A = \sqrt{0.8 g \rho_A} = \sqrt{0.8(9.81)(85)} = \underline{25.8 \text{ m/s}}$

$v_B = \sqrt{0.8 g \rho_B} = \sqrt{0.8(9.81)(200)} = \underline{39.6 \text{ m/s}}$

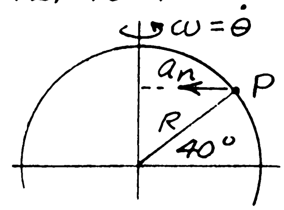
Path BB offers a considerable advantage.

2/109

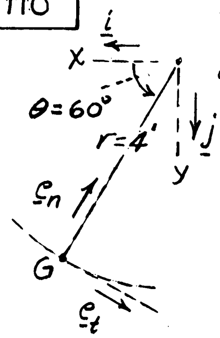
$$a = a_n = r\dot{\theta}^2 = R\cos\gamma\,\dot{\theta}^2$$

$$= \frac{12.742(10^6)}{2}\cos 40^\circ\,(0.729\times10^{-4})^2$$

$$= \underline{0.0259\ m/s^2}$$



## 2/110




$a_n = r\dot{\theta}^2 = 4(2.00)^2 = 16.00$ ft/sec$^2$

$a_t = r\ddot{\theta} = 4(4.025) = 16.10$ ft/sec$^2$

$\underline{a} = \underline{16.00\underline{e}_n + 16.10\underline{e}_t \text{ ft/sec}^2}$

$a_x = -16.00\cos 60° - 16.10\sin 60° = -21.9$ ft/sec$^2$

$a_y = 16.10\cos 60° - 16.00\sin 60° = -5.81$ ft/sec$^2$

$\underline{a} = \underline{-21.9\underline{i} - 5.81\underline{j} \text{ ft/sec}^2}$

2/111

$$a_n = g = \frac{v^2}{\rho} = \frac{[17,369\,(5280/3600)]^2}{(3959+150)(5280)}$$

$$= \underline{29.91\ \text{ft/sec}^2}$$

Check: $g = g_0\left(\dfrac{R}{R+h}\right)^2 = 32.22\left(\dfrac{3959}{3959+150}\right)^2$

$$= \underline{29.91\ \text{ft/sec}^2} \quad \checkmark$$

**2/112**

$a_t = 20/3.6 = 5.56 \text{ m/s}^2$

$a^2 = a_n^2 + a_t^2, \quad a_n^2 = \overline{3(9.81)}^2 - \overline{5.56}^2 = 835.2$

$$a_n = 28.90 \text{ m/s}^2$$

$a_n = v^2/\rho, \quad \rho = \dfrac{(800/3.6)^2}{28.90} = \underline{1709 \text{ m}}$

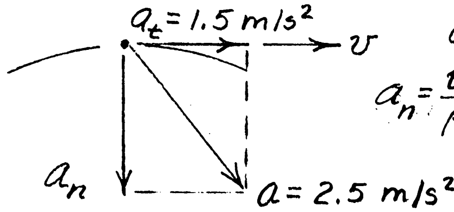
2/113 For $P_1$ $a_n = v^2/r$ , $v = \sqrt{0.1(40)} = \underline{2 \text{ m/s}}$

$$a_1 = \sqrt{a_n^2 + a_t^2} = \sqrt{\overline{40}^2 + \overline{30}^2} = \underline{50 \text{ m/s}^2}$$

For $P_2$ $a_n = v^2/r = 2^2/0.05 = 80 \text{ m/s}^2$

$$a_2 = \sqrt{a_n^2 + a_t^2} = \sqrt{\overline{80}^2 + \overline{30}^2} = \underline{85.4 \text{ m/s}^2}$$

2/114




$$a^2 = a_n^2 + a_t^2$$

$$a_n = \frac{v^2}{\rho} = \sqrt{a^2 - a_t^2}$$

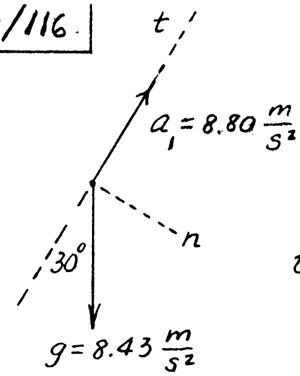
$$v^2 = 200\sqrt{2.5^2 - 1.5^2} = 200(2) = 400\ m^2/s^2$$

$$v = 20\ m/s \quad \text{or} \quad 20(3.6) = \underline{72\ km/h}$$

$\underline{2/115}$ $a_n = v\dot{\beta} = g$, $\dot{\beta} = \dfrac{9.79}{800(10^3)/3600} = 0.04406$ rad/s

or $\dot{\beta} = 0.04406 \dfrac{180}{\pi} = \underline{2.52 \text{ deg/s}}$

2/116




$a_n = g \sin 30° = 8.43(0.5) = 4.22 \text{ m/s}^2$

or $a_n = \frac{4.22}{1000}(3600)^2 = 54\,630 \text{ km/h}^2$

$a_n = \frac{v^2}{\rho}, \quad \rho = \frac{v^2}{a_n} = \frac{(30000)^2}{54630} = \underline{16\,480 \text{ km}}$

$\dot{v} = a_t = 8.80 - 8.43 \cos 30° = \underline{1.499 \text{ m/s}^2}$

2/117 Relative to space station, $a_n = r\dot{\theta}^2$

where $a_n = 32.17$ ft/sec$^2$.

Thus $32.17 = (240 + 20)\,\dot{\theta}^2$, $\dot{\theta} = 0.352\ \frac{\text{rad}}{\text{sec}}$

$N = 0.352\left(\frac{60}{2\pi}\right) = \underline{3.36\ \text{rev/min}}$

2/118 $a_t = \dfrac{v_f - v_i}{\Delta t} = \dfrac{6-3}{2} = 1.5 \text{ m/s}^2$

Halfway through time interval, $v = 4.5$ m/s

$$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\frac{4.5^2}{0.060}\right)^2}$$

$$= \underline{338 \text{ m/s}^2} \quad (34.4g!)$$

$$a_{P_2} = a_t = \underline{1.5 \text{ m/s}^2}$$

2/119 $\underline{r} = \frac{3}{2}t^2\underline{i} + \frac{2}{3}t^3\underline{j}$ in.

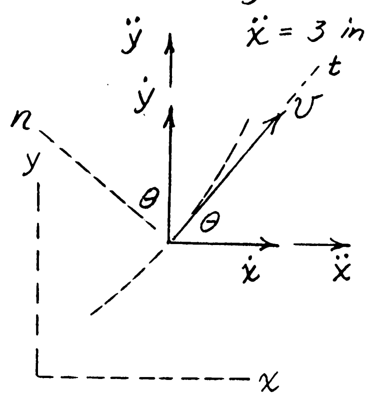
$\underline{v} = \dot{\underline{r}} = 3t\underline{i} + 2t^2\underline{j}$ in./sec

$\underline{a} = \dot{\underline{v}} = 3\underline{i} + 4t\underline{j}$ in./sec$^2$

For $t = 2$ sec, $\dot{x} = 3(2) = 6$ in./sec

$\dot{y} = 2(2^2) = 8$ in./sec

$\ddot{x} = 3$ in./sec$^2$, $\ddot{y} = 8$ in./sec$^2$




$\theta = \tan^{-1}\frac{6}{8} = \tan^{-1}\frac{3}{4}$

$v^2 = \dot{x}^2 + \dot{y}^2 = 6^2 + 8^2 = 100$ (in./sec)$^2$

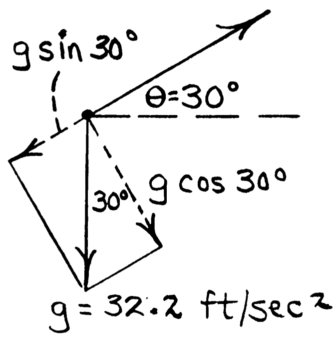
$a_n = \ddot{y}\cos\theta - \ddot{x}\sin\theta$

$= 8(3/5) - 3(4/5) = 12/5$ in./sec$^2$

$a_n = v^2/\rho$, $\rho = \frac{v^2}{a_n} = \frac{100}{12/5}$

$\rho = 41.7$ in.

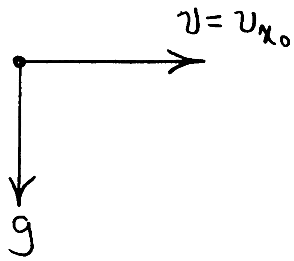
2/120




(a) $a_n = g \cos 30° = \dfrac{v^2}{\rho}$

$$\rho = \frac{100^2}{g \cos 30°} = \underline{359 \text{ ft}}$$

$$\dot{v} = -g \sin 30° = \underline{-16.1 \text{ ft/sec}^2}$$




(b) $a_n = g = \dfrac{v^2}{\rho}$

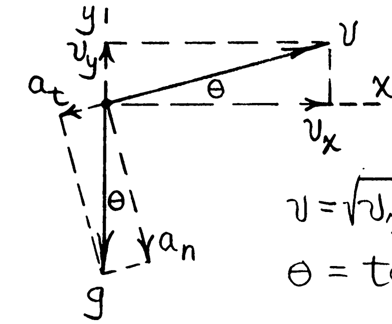
$$\rho = \frac{(100 \cos 30°)^2}{32.2} = \underline{233 \text{ ft}}$$

$$\underline{\dot{v} = 0}$$

2/121 The time $t_{up}$ to apex is found from

$v_y = v_{y_0} - gt$ : $0 = 100 \sin 30° - 32.2\, t_{up}$, $t_{up} = 1.553$ sec

So $t = 1$ sec is before apex and $t = 2.5$ sec is after.




(a) $t = 1$ sec

$v_x = 100 \cos 30° = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(1) = 17.80$ ft/sec

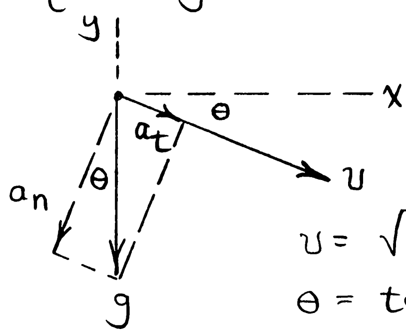
$v = \sqrt{v_x^2 + v_y^2} = 88.4$ ft/sec

$\theta = \tan^{-1} \dfrac{v_y}{v_x} = 11.61°$

$a_n = g \cos\theta = 32.2 \cos 11.61° = 31.5$ ft/sec²

$\rho = \dfrac{v^2}{a_n} = \dfrac{88.4^2}{31.5} = \underline{248 \text{ ft}}$

$a_t = -g \sin\theta = -32.2 \sin 11.61° = \underline{-6.48 \text{ ft/sec}^2}$




(b) $t = 2.5$ sec

$v_x = 86.6$ ft/sec

$v_y = 100 \sin 30° - 32.2(2.5) = -30.5 \dfrac{\text{ft}}{\text{sec}}$

$v = \sqrt{v_x^2 + v_y^2} = 91.8$ ft/sec

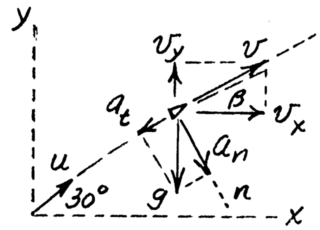
$\theta = \tan^{-1} \dfrac{|v_y|}{v_x} = 19.40°$

$a_n = g \cos\theta = 32.2 \cos 19.40° = 30.4$ ft/sec²

$\rho = \dfrac{v^2}{a_n} = \dfrac{91.8^2}{30.4} = \underline{278 \text{ ft}}$

$a_t = +g \sin\theta = +32.2 \sin 19.40° = \underline{10.70 \text{ ft/sec}^2}$

**2/122** $v = v_0 + at;\quad v_x = 1800(0.866) + 0 = 1559$ ft/sec




$v_y = 1800(0.5) - 32.2(10)$

$= 578$ ft/sec

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\overline{1559}^2 + \overline{578}^2}$

$= 1663$ ft/sec

$\beta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{578}{1559} = 20.3°$

$a_n = g\cos\beta = 32.2(0.9376) = 30.19$ ft/sec$^2$

$a_n = v^2/\rho,\quad \rho = \overline{1663}^2/30.19 = \underline{91{,}600 \text{ ft}}$

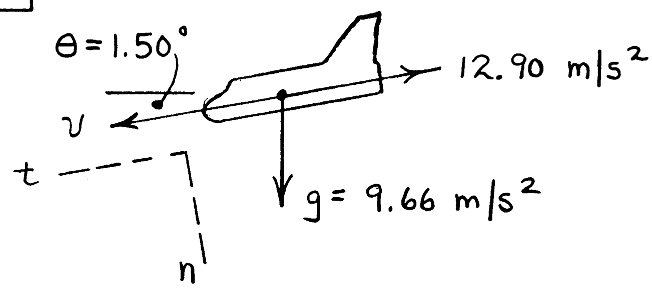
**2/123** $a_n = v^2/\rho = \dfrac{v^2}{R+h}$ where $R = 1738$ km
$h = 200$ km
$g_0 = 1.62$ m/s$^2$ (Appen. B)

$$a_n = g = g_0\left(\frac{R}{R+h}\right)^2 = 1.62\left(\frac{1738}{1738+200}\right)^2 = 1.303 \text{ m/s}^2$$

$$v^2 = (1738+200)\,1.303\,(3600)^2(10^{-3}) = 32.72\,(10^6)\ (\text{km/h})^2$$

$$v = \underline{5720 \text{ km/h}}$$

2/124




$$\dot{v} = a_t = 9.66 \sin 1.50° - 12.90 = \underline{-12.65 \text{ m/s}^2}$$

$$a_n = g \cos\theta = 9.66 \cos 1.5° = 9.657 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{(15\,450/3.6)^2}{9.657}$$

$$\underline{\rho = 1907 \text{ km}}$$

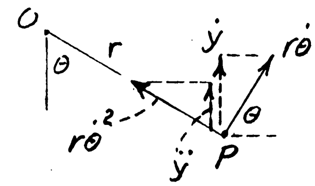
$\boxed{2/125}$ (a) $\dot\theta = \omega,\ \ddot\theta = 0$

$\dot y = r\dot\theta \sin\theta$

$= \underline{r\omega \sin\theta}$

$\ddot y = r\dot\theta^2 \cos\theta$

$= \underline{r\omega^2 \cos\theta}$



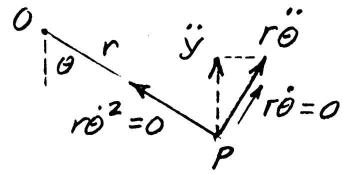

(b) $\dot\theta = 0,\ \ddot\theta = \alpha$

$\dot y = r\dot\theta \sin\theta$

$\underline{= 0}$

$\ddot y = r\ddot\theta \sin\theta$

$= \underline{r\alpha \sin\theta}$



$\boxed{2/126}$ $a_n = 0.8g = \frac{v^2}{\rho} \Rightarrow v = \sqrt{0.8g\rho}$

Car A : $v_A = \sqrt{0.8(9.81)(88)} = 26.3$ m/s

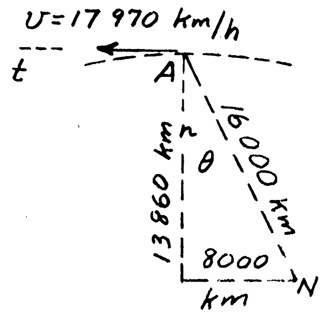
Car B : $v_B = \sqrt{(0.8)(9.81)72} = 23.8$ m/s

$$t_A = \frac{s_A}{v_A} = \frac{\pi(88)}{26.3} = \underline{10.52 \text{ s}}$$

$$t_B = \frac{s_B}{v_A} = \frac{\pi(72) + 2(16)}{23.8} = \underline{10.86 \text{ s}}$$

Car A will win the race!

2/127




Accel. directed from A to N is

$$a = g = g_0\left(\frac{R}{R+h}\right)^2$$

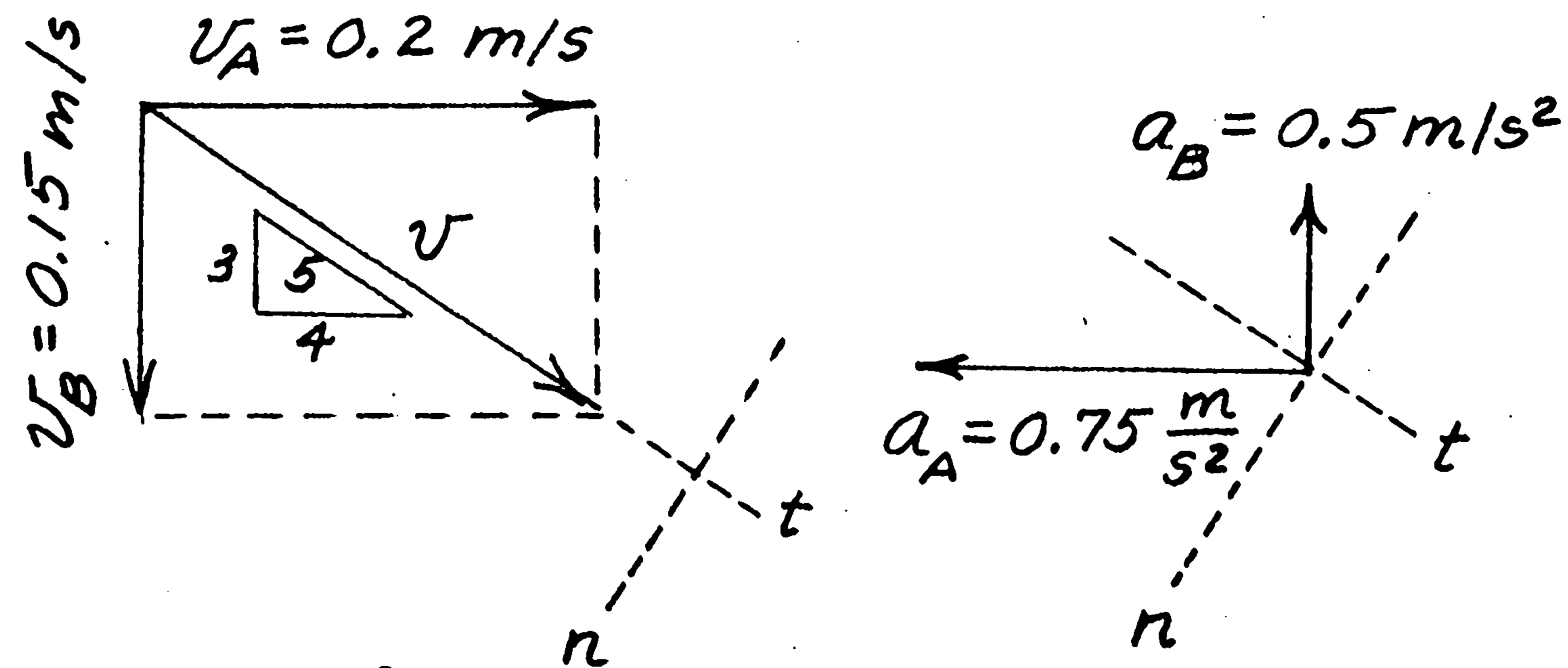
$$= 9.821\left(\frac{6371}{16000}\right)^2 = 1.557 \text{ m/s}^2$$

$$a_n = 1.557 \cos\theta = 1.557(0.866)$$

$$= 1.348 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho},\ \rho = \frac{(17\,970)^2}{1.348(10^{-3})(3600)^2} = \underline{18\,480 \text{ km}}$$

▸2/128 $v = \sqrt{0.15^2 + 0.2^2} = 0.25$ m/s (direction of path)




$a_n = 0.75\left(\frac{3}{5}\right) - 0.50\left(\frac{4}{5}\right) = 0.05 \text{ m/s}^2$

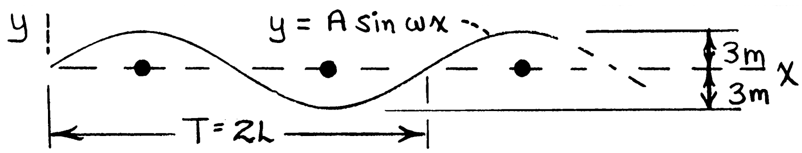
$a_n = v^2/\rho\,,\quad \rho = \overline{0.25}^2/0.05 = \underline{1.25 \text{ m}}$

$a_t = -0.75\left(\frac{4}{5}\right) - 0.5\left(\frac{3}{5}\right) = -0.9 \text{ m/s}^2$

$a_t = \frac{d}{dt}(v) = \frac{d}{dt}(\rho\dot\theta) = \dot\rho\dot\theta + \rho\ddot\theta = \dot\rho\frac{v}{\rho} + \rho\ddot\theta$

so $\dot\rho$ cannot be found until $\ddot\theta$ is known

▶2/129




$y = A \sin \omega x$, where $A = 3$ m & $\omega = \frac{2\pi}{T}$

Radius of curvature $\rho = \dfrac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$\dfrac{dy}{dx} = A\omega \cos \omega x$, $\dfrac{d^2y}{dx^2} = -A\omega^2 \sin \omega x$

Set $\dfrac{d\rho}{dx} = 0$ to show that $|\rho|$ is a min @ $x = \frac{T}{4}$ & $x = \frac{3T}{4}$.

$$\rho_{min} = \frac{\left[1 + \left\{A \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)\right\}^2\right]}{+A\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right)} = \frac{T^2}{4\pi^2 A}$$

$a_n = \dfrac{v^2}{\rho}$ : $0.7\,(9.81) = \dfrac{(80/3.6)^2}{T^2/(4\pi^2 \cdot 3)}$

$T = 92.3$ m $= 2L$, $\underline{L = 46.1 \text{ m}}$

**▶ 2/130**

$$\frac{dy}{dx} = 3x^{1/2},\quad \frac{d^2y}{dx^2} = \frac{3}{2}x^{-1/2},\quad \rho = \frac{[1+9x]^{3/2}}{\frac{3}{2}x^{-1/2}} = \frac{2}{3}\sqrt{x(1+9x)^3}$$

$$s = \int ds = \int_0^x \sqrt{1+(dy/dx)^2}\,dx = \int_0^x \sqrt{1+9x}\,dx = \frac{2}{27}\left[\sqrt{(1+9x)^3} - 1\right]$$

When $t = 1$ sec, $s = 2$ in., $2 = \frac{2}{27}\left[\sqrt{(1+9x)^3} - 1\right]$, $x = 0.913$ in.

and $\rho = \frac{2}{3}\sqrt{0.913\,(1 + 9\times 0.913)^3} = 17.84$ in.

$v = \dot{s} = 6t^2 = 6(1^2) = 6$ in./sec, $a_n = \frac{v^2}{\rho} = \frac{6^2}{17.84} = 2.02$ in./sec$^2$

$a_t = \ddot{s} = 12t = 12(1) = 12$ in./sec$^2$

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(2.02)^2 + 12^2} = \underline{12.17 \text{ in./sec}^2}$

2/131

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta = 1.5\,\underline{e}_r + (24+7)\left(5\tfrac{\pi}{180}\right)\underline{e}_\theta$$

$$= \underline{1.5\,\underline{e}_r + 2.71\,\underline{e}_\theta \quad \text{ft/sec}}$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= \left[-4 - 31\left(5\tfrac{\pi}{180}\right)^2\right]\underline{e}_r + \left[31\left(2\tfrac{\pi}{180}\right) + 2(1.5)\left(5\tfrac{\pi}{180}\right)\right]\underline{e}_\theta$$

$$= \underline{-4.24\,\underline{e}_r + 1.344\,\underline{e}_\theta \quad \text{ft/sec}^2}$$

2/132

| Position | $r$ | $\dot{r}$ | $\ddot{r}$ | $\theta$ | $\dot{\theta}$ | $\ddot{\theta}$ |
|---|---|---|---|---|---|---|
| A | + | − | + | + | + | + |
| B | + | 0 | + | + | + | 0 |
| C | + | + | + | + | + | − |

Notes : (1) $r \geq 0$ , always , by definition

(2) $\dot{r}$ determined by inspection

(3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$

(4) $\theta \geq 0$ , by definition in figure

(5) $\dot{\theta} > 0$ here , by inspection

(6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

2/133 $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 200(8^2) = -12\,800 \text{ mm/s}^2$

or $\underline{a_r = -12.80 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 200(-20) + 2(-300)(8) = -8800 \text{ m/s}^2$

or $\underline{a_\theta = -8.80 \text{ m/s}^2}$

2/134

A $\underline{v}_A = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta = \underline{v\,\underline{e}_r + l\Omega\,\underline{e}_\theta}$

$\underline{a}_A = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underline{e}_\theta$

$= \underline{-l\Omega^2\,\underline{e}_r + 2v\Omega\,\underline{e}_\theta}$

B $\underline{v}_B = \underline{4v\,\underline{e}_r + 2l\Omega\,\underline{e}_\theta}$

$\underline{a}_B = \underline{-2l\Omega^2\,\underline{e}_r + 8v\Omega\,\underline{e}_\theta}$

2/135 $r = 375 + 125 = 500 \text{ mm}$, $\dot{r} = \dot{l} = -150 \frac{\text{mm}}{\text{s}}$

$\ddot{r} = 0$, $\dot{\theta} = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3} \text{ rad/s}$, $\ddot{\theta} = 0$

$v_r = \dot{r} = -150 \frac{\text{mm}}{\text{s}}$, $v_\theta = r\dot{\theta} = 500\left(\frac{\pi}{3}\right) = 524 \frac{\text{mm}}{\text{s}}$

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545 \frac{\text{mm}}{\text{s}}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500\left(\frac{\pi}{3}\right)^2 = -548 \text{ mm/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150)(\pi/3) = -314 \text{ mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632 \text{ mm/s}^2}$

2/136 From $\underline{a} = [\ddot{r} - r\dot{\theta}^2]\underline{e}_r + [r\ddot{\theta} + 2\dot{r}\dot{\theta}]\underline{e}_\theta$

we have, for $\ddot{r} = \ddot{\theta} = 0$, $\dot{\theta} = \Omega$, and $r = l$:

$$a = \sqrt{(l\Omega^2)^2 + (2\dot{l}\Omega)^2} = \Omega\sqrt{l^2\Omega^2 + 4\dot{l}^2}$$

$$0.011 = 0.05\sqrt{[4.2(0.05)]^2 + 4\dot{l}^2}$$

Solve for $\dot{l}$: $\underline{\dot{l} = 0.0328 \text{ m/s}}$

or $\underline{\dot{l} = 32.8 \text{ mm/s}}$

**2/137** $r = t^3/3, \quad \dot{r} = t^2, \quad \ddot{r} = 2t$

$$\theta = 2\cos\frac{\pi t}{6}, \quad \dot{\theta} = -\frac{\pi}{3}\sin\frac{\pi t}{6}, \quad \ddot{\theta} = -\frac{\pi^2}{18}\cos\frac{\pi t}{6}$$

For $t = 2$ s, $r = 8/3$ m, $\dot{r} = 4$ m/s, $\ddot{r} = 4$ m/s$^2$

$$\dot{\theta} = -\frac{\pi}{3}\sin\frac{\pi}{3} = -\frac{\pi}{2\sqrt{3}} = -0.907 \text{ rad/s}$$

$$\ddot{\theta} = -\frac{\pi^2}{18}\cos\frac{\pi}{3} = -\frac{\pi^2}{36} = -0.274 \text{ rad/s}^2$$

$v_r = \dot{r} = 4$ m/s, $v_\theta = r\dot{\theta} = (8/3)(-0.907) = -2.42$ m/s

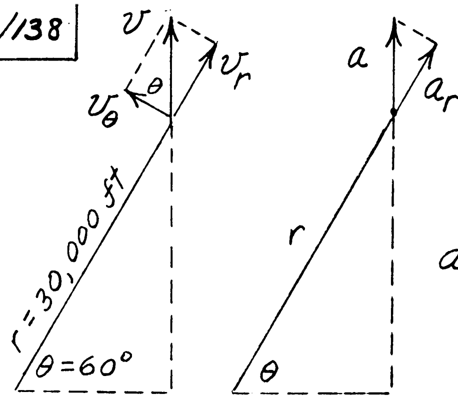
$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - (8/3)(-0.907)^2 = 4 - 2.19 = 1.807$ m/s$^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (8/3)(-0.274) + 2(4)(-0.907) = -7.99$ m/s$^2$

Thus $\underline{\mathbf{v} = 4\mathbf{e}_r - 2.42\mathbf{e}_\theta \text{ m/s}}$

$\underline{\mathbf{a} = 1.807\mathbf{e}_r - 7.99\mathbf{e}_\theta \text{ m/s}^2}$

2/138




$v_\theta = r\dot{\theta} = 30(10^3)(0.020)$
$= 600$ ft/sec

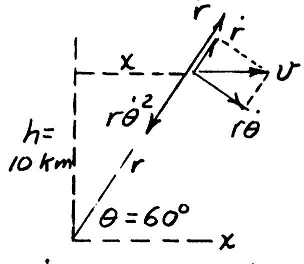
$v = v_\theta / \cos 60°$
$= 600/0.5 = \underline{1200 \text{ ft/sec}}$

$a_r = \ddot{r} - r\dot{\theta}^2$
$= 70 - 30(10^3)(0.020)^2$
$= 58$ ft/sec$^2$

$a = a_r / \sin 60°$
$= 58/0.866 = \underline{67.0 \text{ ft/sec}^2}$

2/139 $a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{1}{1/3}\left(-\frac{20-12}{7-4}\right) = -3\,(8/3) = \underline{-8\ m/s^2}$

**2/140** Acceleration in all directions is zero, so




$\dot{\theta} = -0.020$ rad/s

$a_r = \ddot{r} - r\dot{\theta}^2 = 0, \quad \ddot{r} = r\dot{\theta}^2$

$r = h/\sin\theta, \quad r = \frac{10}{\sqrt{3}/2} = 11.55$ km

$\ddot{r} = 11.55\,(-0.020)^2 = 0.00462$ km/s$^2$

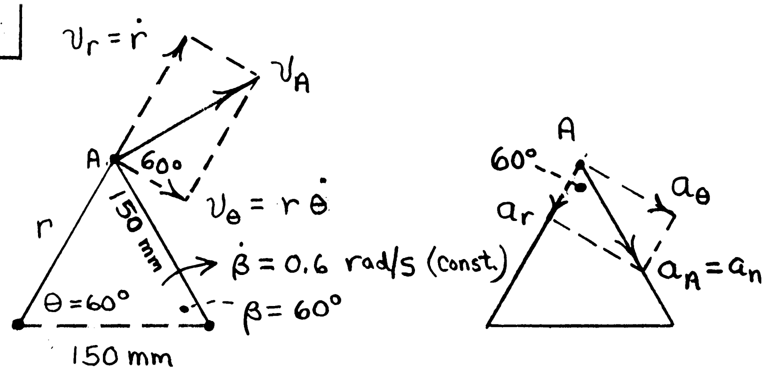
$\underline{= 4.62 \text{ m/s}^2}$

$v = |r\dot{\theta}|/\sin\theta = h\dot{\theta}/\sin^2\theta$

$= \frac{|10(-0.020)|}{(\sqrt{3}/2)^2} = 0.267$ km/s

or $v = 0.267(3600) = \underline{960 \text{ km/h}}$

2/141




For $\beta = 60°$, $\theta = 60°$, $r = 150$ mm

$v_A = 150(0.6) = 90$ mm/s

$v_\theta = r\dot{\theta} = -v_A \cos 60°$, $\dot{\theta} = \dfrac{-90 \cos 60°}{150} = \underline{-0.3 \ \dfrac{\text{rad}}{\text{s}}}$

$v_r = \dot{r} = v_A \sin 60° = 90 \sin 60° = \underline{77.9 \text{ mm/s}}$

$a_A = a_n = 150(0.6)^2 = 54$ mm/s$^2$

$a_r = \ddot{r} - r\dot{\theta}^2 : -54 \cos 60° = \ddot{r} - 150(-0.3)^2$

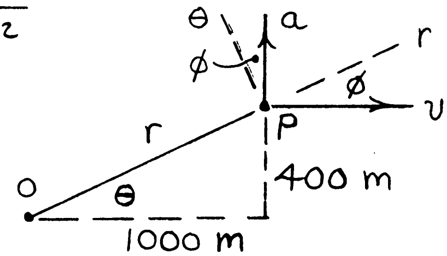
$\underline{\ddot{r} = -13.5 \text{ mm/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : -54 \sin 60° = 150\ddot{\theta} + 2(77.9)(-0.3)$

$\underline{\ddot{\theta} = 0}$

2/142 $r = \sqrt{1000^2 + 400^2}$

$= 1077 \text{ m}$




$\theta = \tan^{-1}\frac{400}{1200} = 21.8°$

$v = \frac{600}{3.6} = 166.7 \text{ m/s}$

$a = a_n = \frac{v^2}{\rho} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$

$v_r = \dot{r} = v\cos\theta = 166.7\cos 21.8° = 154.7 \text{ m/s}$

$v_\theta = r\dot{\theta}$ : $-166.7\sin 21.8° = 1077\dot{\theta}$, $\dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $23.1\sin 21.8° = \ddot{r} - 1077(-0.0575)^2$

$\underline{\ddot{r} = 12.15 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$ : $23.1\cos 21.8° = 1077\ddot{\theta} + 2(154.7)(-0.0575)$

$\underline{\ddot{\theta} = 0.0365 \text{ rad/s}^2}$

**2/143** $a_r = \ddot{r} - r\dot{\theta}^2$ where for $\theta = 0$, $v = r\dot{\theta}$

Also for $\theta = 0$, $a_r = -a_n$ where $(a_n = v^2/\rho)$

So $\ddot{r} - r(v/r)^2 = -v^2/\rho$, $\ddot{r} = v^2\left(\frac{1}{r} - \frac{1}{\rho}\right) = \underline{-v^2\left(\frac{1}{\rho} - \frac{1}{r}\right)}$

2/144 $\theta = 0.4 + 0.12t + 0.06t^3$ | $r = 0.8 - 0.1t - 0.05t^2$

$\dot{\theta} = 0.12 + 0.18t^2$ | $\dot{r} = -0.1 - 0.1t$

$\ddot{\theta} = 0.36t$ | $\ddot{r} = -0.1$

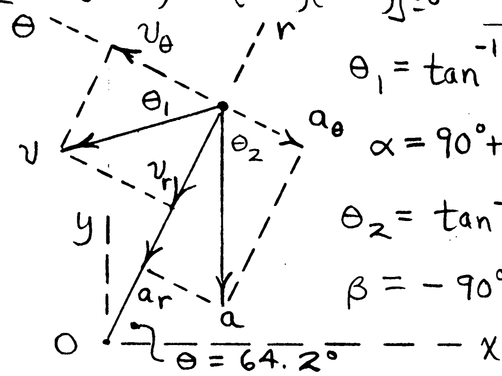
At $t = 2$ s: $\begin{cases} \theta = 1.12 \text{ rad} & r = 0.4 \text{ m} \\ \dot{\theta} = 0.84 \text{ rad/s} & \dot{r} = -0.3 \text{ m/s} \\ \ddot{\theta} = 0.72 \text{ rad/s}^2 & \ddot{r} = -0.1 \text{ m/s}^2 \end{cases}$

$$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.3\underline{e}_r + 0.4(0.84)\underline{e}_\theta$$

$$= \underline{-0.3\underline{e}_r + 0.336\underline{e}_\theta \text{ m/s}}$$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta = [-0.1 - 0.4(0.84)^2]\underline{e}_r$$

$$+ [0.4(0.72) + 2(-0.3)(0.84)]\underline{e}_\theta = \underline{-0.382\underline{e}_r - 0.216\underline{e}_\theta \text{ m/s}^2}$$




$\theta_1 = \tan^{-1}\dfrac{|v_r|}{|v_\theta|} = 41.8°$

$\alpha = 90° + \theta + \theta_1 = \underline{195.9°}$

$\theta_2 = \tan^{-1}\dfrac{|a_r|}{|a_\theta|} = 60.5°$

$\beta = -90° + \theta - \theta_2 = \underline{-86.4°}$

**2/145** $\theta = 0.8t - \frac{t^2}{20}$ rad $\qquad r = 1.6 - 0.2t$ m

$\dot{\theta} = 0.8 - 0.1t$ rad/s $\qquad \dot{r} = -0.2$ m/s

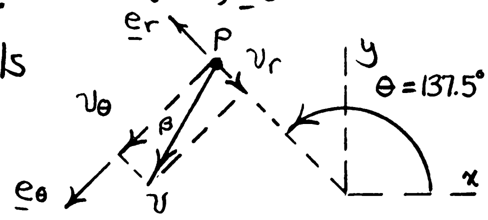
$\ddot{\theta} = -0.1$ rad/s$^2$ $\qquad \ddot{r} = 0$

At $t = 4$ s : $\theta = 2.4$ rad $(137.5°)$, $\dot{\theta} = 0.4$ rad/s

$r = 0.8$ m

$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.2\underline{e}_r + 0.8(0.4)\underline{e}_\theta$

$= -0.2\,\underline{e}_r + 0.32\,\underline{e}_\theta$ m/s




$\underline{v = 0.377 \text{ m/s}}$

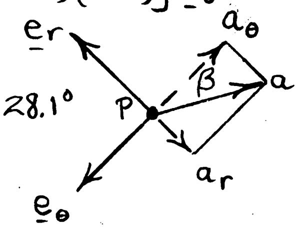
$\beta = \tan^{-1}\left(\frac{0.2}{0.32}\right) = 32.0°$

$\alpha = \theta + 90° + \beta = \underline{260°}$

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [0 - 0.8(0.4)^2]\underline{e}_r + [0.8(-0.1) + 2(-0.2)(0.4)]\underline{e}_\theta$

$= -0.1280\,\underline{e}_r - 0.240\,\underline{e}_\theta$ m/s$^2$




$\underline{a = 0.272 \text{ m/s}^2}$, $\beta = \tan^{-1}\frac{0.1280}{0.240} = 28.1°$

$\alpha = \theta - 90 + \beta = \underline{19.44°}$

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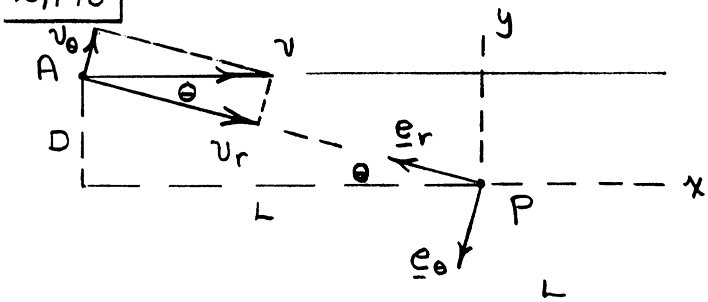


$$v' = |v_r| = v\cos\theta = v\frac{L}{\sqrt{L^2+D^2}}$$

Numbers: $v' = 70\,\dfrac{500}{\sqrt{500^2+20^2}} = \underline{69.9\ \text{mi/hr}}$

The factor of $\cos\theta$ is the basis for the statement that, kinematically, radar can yield an accurate or low, but not high, speed measurement. As can be seen, however, adherence to the speed limit (not reliance upon $\cos\theta$) is the best policy!

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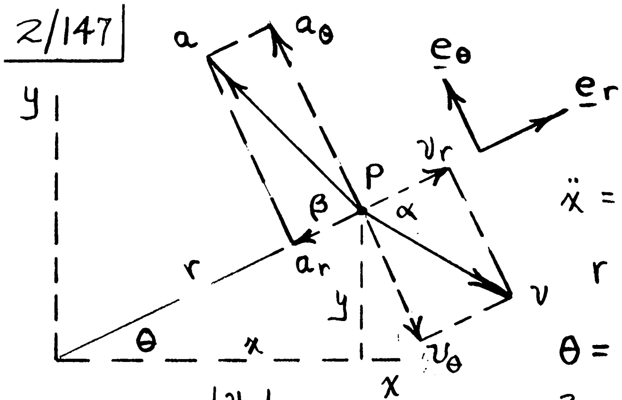


$x = 4\text{ m}, \quad y = 2\text{ m}$

$\dot{x} = 2\sqrt{3}\text{ m/s}, \quad \dot{y} = -2\ \frac{\text{m}}{\text{s}}$

$\ddot{x} = -5\text{ m/s}^2, \quad \ddot{y} = 5\ \frac{\text{m}}{\text{s}^2}$

$r = \sqrt{x^2+y^2} = \underline{2\sqrt{5}\text{ m}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \underline{26.6^\circ}$

$$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} + \theta = \tan^{-1}\frac{2}{2\sqrt{3}} + 26.6^\circ = 56.6^\circ$$

$$\beta = \tan^{-1}\left|\frac{a_y}{a_x}\right| + \theta = \tan^{-1}\frac{5}{5} + 26.6^\circ = 71.6^\circ$$

$$v = \sqrt{v_y^2 + v_x^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4\text{ m/s}$$

$$a = \sqrt{a_y^2 + a_x^2} = \sqrt{5^2 + 5^2} = 7.07\text{ m/s}^2$$

$$\dot{r} = v_r = v\cos\alpha = 4\cos 56.6^\circ = \underline{2.20\text{ m/s}}$$

$$v_\theta = -v\sin\alpha = -4\sin 56.6^\circ = -3.34\text{ m/s}$$

$$v_\theta = r\dot{\theta} : -3.34 = 2\sqrt{5}\,\dot{\theta}, \quad \underline{\dot{\theta} = -0.746\text{ rad/s}}$$

$$a_r = -a\cos\beta = -7.07\cos 71.6^\circ = -2.24\text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 : -2.24 = \ddot{r} - 2\sqrt{5}(0.746)^2, \quad \underline{\ddot{r} = 0.255\ \frac{\text{m}}{\text{s}^2}}$$

$$a_\theta = a\sin\beta = 7.07\sin 71.6^\circ = 6.71\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 6.71 = 2\sqrt{5}\,\ddot{\theta} + 2(2.20)(-0.746), \quad \underline{\ddot{\theta} = 2.24\ \frac{\text{rad}}{\text{s}^2}}$$

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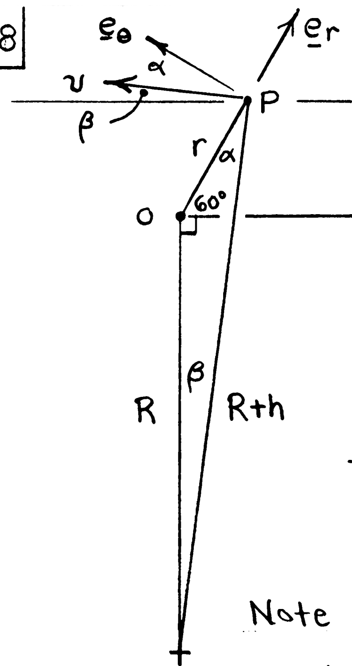


$$\frac{\sin 150°}{R+h} = \frac{\sin \alpha}{R}$$

$$\frac{\sin 150°}{3959+150} = \frac{\sin \alpha}{3959}$$

$$\alpha = 28.8°$$

$$\alpha + \beta + 150° = 180° \Rightarrow \beta = 1.200°$$

$$v_r = \dot{r} = -12,272 = -v \sin \alpha$$

$$-12,272 = -v \sin 28.8°$$

$$v = \underline{25,474 \text{ ft/sec}}$$

Note: Because $\underline{v}$ is nearly parallel to the horizontal at O ($\beta = 1.200°$), one obtains a close approximation ($v = 24,544 \frac{\text{ft}}{\text{sec}}$) to the correct answer by neglecting $\beta$ (assuming a flat earth).

2/149 $r = K\theta$, $\dot{r} = K\dot{\theta}$, $\ddot{r} = K\ddot{\theta} = K\alpha$

$$\dot{\theta}^2 = 2\int_{\pi/4}^{3\pi/4} \alpha\, d\theta = 2\alpha\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = \alpha\pi$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = K\alpha - K\theta(\alpha\pi) = K\alpha\left(1 - \frac{3\pi^2}{4}\right)$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = K\frac{3\pi}{4}\alpha + 2K\sqrt{\alpha\pi}\sqrt{\alpha\pi} = \frac{11}{4}K\alpha\pi$$

$$a = \sqrt{a_r^2 + a_\theta^2} = K\alpha\sqrt{\left(1 - \frac{3\pi^2}{4}\right)^2 + \left(\frac{11}{4}\pi\right)^2} = K\alpha\sqrt{1 + \frac{97}{16}\pi^2 + \frac{9}{16}\pi^4}$$

$$= \underline{10.76\, K\alpha}$$

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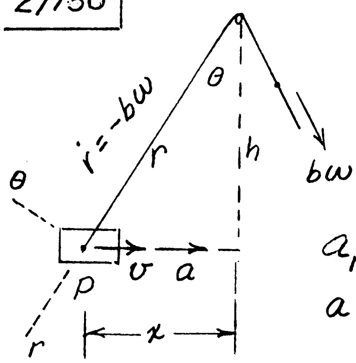


$v \sin\theta = -\dot{r} = b\omega \quad , \quad \ddot{r} = 0$

$\underline{v = b\omega / \sin\theta}$

$v_\theta = r\dot{\theta} = -v\cos\theta = -b\omega \operatorname{ctn}\theta$

$\dot{\theta} = -\dfrac{b\omega}{r}\operatorname{ctn}\theta$

$a_r = \ddot{r} - r\dot{\theta}^2 = -a\sin\theta$

$a = \dfrac{1}{\sin\theta}\left(r\dot{\theta}^2 - \ddot{r}\right) = \underline{\dfrac{b^2\omega^2}{h}\operatorname{ctn}^3\theta}$

Alternatively: $x^2 + h^2 = r^2, \quad 2x\dot{x} + 0 = 2r\dot{r}$

$v = -\dot{x} = -\dfrac{r\dot{r}}{x} = \dfrac{b\omega}{\sin\theta}$ (check)

$x\ddot{x} + \dot{x}^2 = \dot{r}^2 + 0$

$a = -\ddot{x} = \dfrac{1}{x}\left(\dot{x}^2 - \dot{r}^2\right) = \dfrac{1}{h\tan\theta}\left(\dfrac{b^2\omega^2}{\sin^2\theta} - b^2\omega^2\right)$

$= \dfrac{b^2\omega^2}{h}\left[\dfrac{1}{\tan\theta}\,\dfrac{\cos^2\theta}{\sin^2\theta}\right] = \dfrac{b^2\omega^2}{h}\operatorname{ctn}^3\theta$ (check)

$\underline{2/151}$ $r = r_0 + b_0 \sin 2\pi n t, \quad \dot{r} = 2\pi n b_0 \cos 2\pi n t$

$$\ddot{r} = -4\pi^2 b_0 n^2 \sin 2\pi n t$$

$$\dot{\theta} = \omega, \quad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_r = -4\pi^2 b_0 n^2 \sin 2\pi n t - (r_0 + b_0 \sin 2\pi n t)\omega^2$$

$$= -(4\pi^2 n^2 + \omega^2) b_0 \sin 2\pi n t - r_0 \omega^2$$

$$|a_r|_{max} = \underline{(4\pi^2 n^2 + \omega^2) b_0 + r_0 \omega^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad a_\theta = 0 + 4\pi b_0 n \omega \cos 2\pi n t$$

$$|a_\theta|_{max} = \underline{4\pi b_0 n \omega}$$

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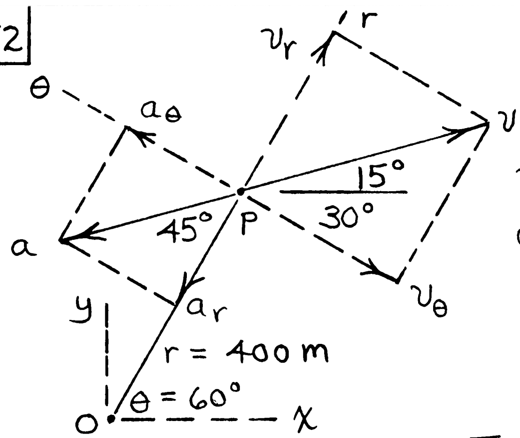


$v = \frac{90}{3.6} = 25$ m/s

$a = 0.5$ m/s$^2$

$v_r = \dot{r} = v \sin 45° = 25 \frac{\sqrt{2}}{2} = \underline{17.68 \text{ m/s}}$

$v_\theta = r\dot{\theta} : -25 \cos 45° = 400\dot{\theta}, \quad \underline{\dot{\theta} = -0.0442 \text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 : -0.5 \cos 45° = \ddot{r} - 400(-0.0442)^2$

$$\underline{\ddot{r} = 0.428 \text{ m/s}^2}$$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0.5 \sin 45° = 400\ddot{\theta} + 2(17.68)(-0.0442)$

$$\underline{\ddot{\theta} = 0.00479 \text{ rad/s}^2}$$

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$$\begin{cases} r = 0.75 + 0.5 = 1.25\text{ m} & \theta = 30° \\ \dot{r} = 0.2\text{ m/s} & \dot{\theta} = 0.1745\ \frac{\text{rad}}{\text{s}} \\ \ddot{r} = -0.3\text{ m/s}^2 & \ddot{\theta} = 0 \end{cases}$$

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\,\underline{e}_r + 0.218\underline{e}_\theta\ \frac{\text{m}}{\text{s}}$$

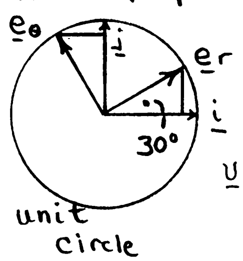
$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296\text{ m/s}}$$

$$\underline{a} = a_r\underline{e}_r + a_\theta\underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= [-0.3 - 1.25(0.1745)^2]\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\underline{e}_\theta$$

$$= -0.338\underline{e}_r + 0.0698\,\underline{e}_\theta\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345\text{ m/s}^2}$$




$$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$$

$$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$$

$$\underline{v} = 0.2[\underline{i}\,c30° + \underline{j}\,s30°] + 0.218[-\underline{i}\,s30° + \underline{j}\,c30°]$$

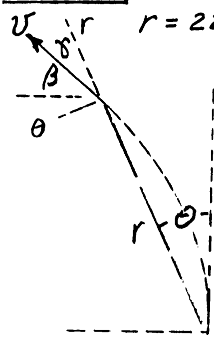
$$= \underline{0.064\underline{i} + 0.289\underline{j}\ \text{m/s}}$$

$$\underline{a} = -0.338[\underline{i}\,c30° + \underline{j}\,s30°] + 0.0698[-\underline{i}\,s30° + \underline{j}\,c30°]$$

$$= \underline{-0.328\,\underline{i} - 0.1086\underline{j}\ \text{m/s}^2}$$

$\boxed{2/154}$ $\theta = 22°$, $\dot{\theta} = 0.0788$ rad/s, $\ddot{\theta} = -0.0341$ rad/s$^2$

$r = 2200$ m, $\dot{r} = 500$ m/s, $\ddot{r} = 4.66$ m/s$^2$




$v_r = \dot{r}$, $v_\theta = r\dot{\theta} = 2200(0.0788) = 173.4$ m/s

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(500)^2 + (173.4)^2} = \underline{529 \text{ m/s}}$

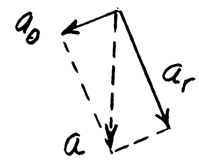
$\gamma = \tan^{-1} v_\theta / v_r = \tan^{-1}\dfrac{173.4}{500} = 19.12°$

$\beta = 90 - \gamma - \theta = 90 - 19.12 - 22 = \underline{48.9°}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 4.66 - 2200(0.0788)^2 = -9.00$ m/s$^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2200(-0.0341) + 2(500)(0.0788) = 3.78$ m/s$^2$

$a = \sqrt{a_\theta^2 + a_r^2} = \sqrt{(3.78)^2 + (9.00)^2} = \underline{9.76 \text{ m/s}^2}$



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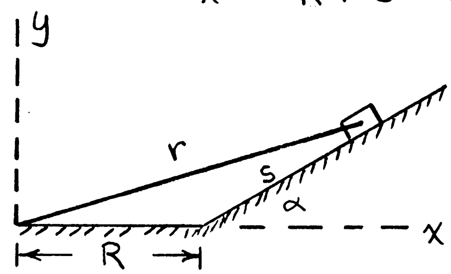


$$x = R + s\cos\alpha = R + (s_0 + v_0 t + \tfrac{1}{2}at^2)\cos\alpha$$

$$= R + \tfrac{1}{2}at^2\cos\alpha$$

$$y = s\sin\alpha$$

$$= \tfrac{1}{2}at^2\sin\alpha$$

$$r = \sqrt{x^2+y^2} = \sqrt{(R+\tfrac{1}{2}at^2\cos\alpha)^2 + (\tfrac{1}{2}at^2\sin\alpha)^2}$$

$$= \sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}$$

$$\dot{r} = \tfrac{1}{2}(R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4)^{-1/2}\left[2Rat\cos\alpha + a^2t^3\right]$$

$$= \underline{\frac{\tfrac{1}{2}at(2R\cos\alpha + at^2)}{\sqrt{R^2 + Rat^2\cos\alpha + \tfrac{1}{4}a^2t^4}}}$$

2/156 From the solution to Prob. 2/155:

$$\begin{cases} x = R + \frac{1}{2}at^2 \cos\alpha \\ y = \frac{1}{2}at^2 \sin\alpha \end{cases}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\frac{1}{2}at^2 \sin\alpha}{R + \frac{1}{2}at^2\cos\alpha}\right]$$

$$\dot{\theta} = \frac{\dfrac{(R+\frac{1}{2}at^2\cos\alpha)(at\sin\alpha) - (\frac{1}{2}at^2\sin\alpha)(at\cos\alpha)}{(R+\frac{1}{2}at^2\cos\alpha)^2}}{1 + \left[\dfrac{\frac{1}{2}at^2\sin\alpha}{R+\frac{1}{2}at^2\cos\alpha}\right]^2}$$

Simplify to

$$\dot{\theta} = \frac{Rat\sin\alpha}{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}$$

**2/157** $r = 1.6 + 0.3\sin\frac{\pi t}{2}$ m ; $\theta = \frac{\pi}{4} + \frac{\pi}{8}\sin\frac{\pi t}{2}$

$\dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$ m/s ; $\dot{\theta} = \frac{\pi^2}{16}\cos\frac{\pi t}{2}$ rad/s

$\ddot{r} = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2}$ m/s$^2$ ; $\ddot{\theta} = -\frac{\pi^3}{32}\sin\frac{\pi t}{2}$ rad/s$^2$

$$v_r = \dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$$

$$v_\theta = r\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2} - \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(1.6 + 0.3\sin\frac{\pi t}{2}\right)\left(-\frac{\pi^3}{32}\sin\frac{\pi t}{2}\right) + 2\left(\frac{0.3\pi}{2}\cos\frac{\pi t}{2}\right)\left(\frac{\pi^2}{16}\cos\frac{\pi t}{2}\right)$$

At $t = 1$ s: $\left.\begin{matrix} v_r = 0 \\ v_\theta = 0 \end{matrix}\right\}$ $v = \sqrt{v_r^2 + v_\theta^2} = \underline{0}$

$\left.\begin{matrix} a_r = -0.740 \text{ m/s}^2 \\ a_\theta = -1.841 \text{ m/s}^2 \end{matrix}\right\}$ $a = \sqrt{0.740^2 + 1.841^2} = \underline{1.984 \text{ m/s}^2}$

At $t = 2$ s: $\left.\begin{matrix} v_r = -0.471 \text{ m/s} \\ v_\theta = 0.987 \text{ m/s} \end{matrix}\right\}$ $\underline{v = 1.094 \text{ m/s}}$

$\left.\begin{matrix} a_r = -0.609 \text{ m/s}^2 \\ a_\theta = 0.581 \text{ m/s}^2 \end{matrix}\right\}$ $\underline{a = 0.842 \text{ m/s}^2}$

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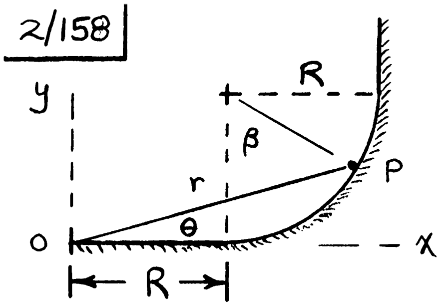



$s = R + R\beta = vt$ :

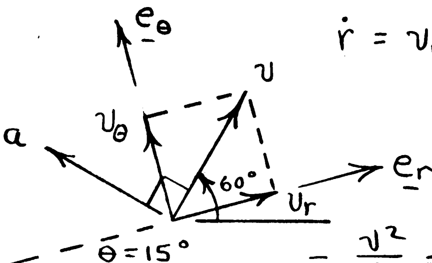
$1.2 + 1.2\beta = 0.6\left(2\left(1+\frac{\pi}{3}\right)\right)$

$\beta = \frac{\pi}{3}$ or $60°$

$x = R + R\sin\beta = 1.2 + 1.2\sin 60° = 2.24$ m

$y = R - R\cos\beta = 1.2 - 1.2\cos 60° = 0.6$ m

$r = \sqrt{x^2+y^2} = \underline{2.32 \text{ m}}$, $\theta = \tan^{-1}\frac{y}{x} = \underline{15°}$




$\dot{r} = v_r = v\cos 45° = 0.6\frac{\sqrt{2}}{2} = \underline{0.424 \frac{m}{s}}$

$v_\theta = r\dot{\theta} = 2.32\dot{\theta} = 0.6\frac{\sqrt{2}}{2}$

$\underline{\dot{\theta} = 0.1830 \text{ rad/s}}$

$a = \frac{v^2}{\rho} = \frac{0.6^2}{1.2} = 0.3 \text{ m/s}^2$

$a_r = -a\frac{\sqrt{2}}{2} = \ddot{r} - r\dot{\theta}^2 : -0.3\frac{\sqrt{2}}{2} = \ddot{r} - 2.32(0.1830)^2$

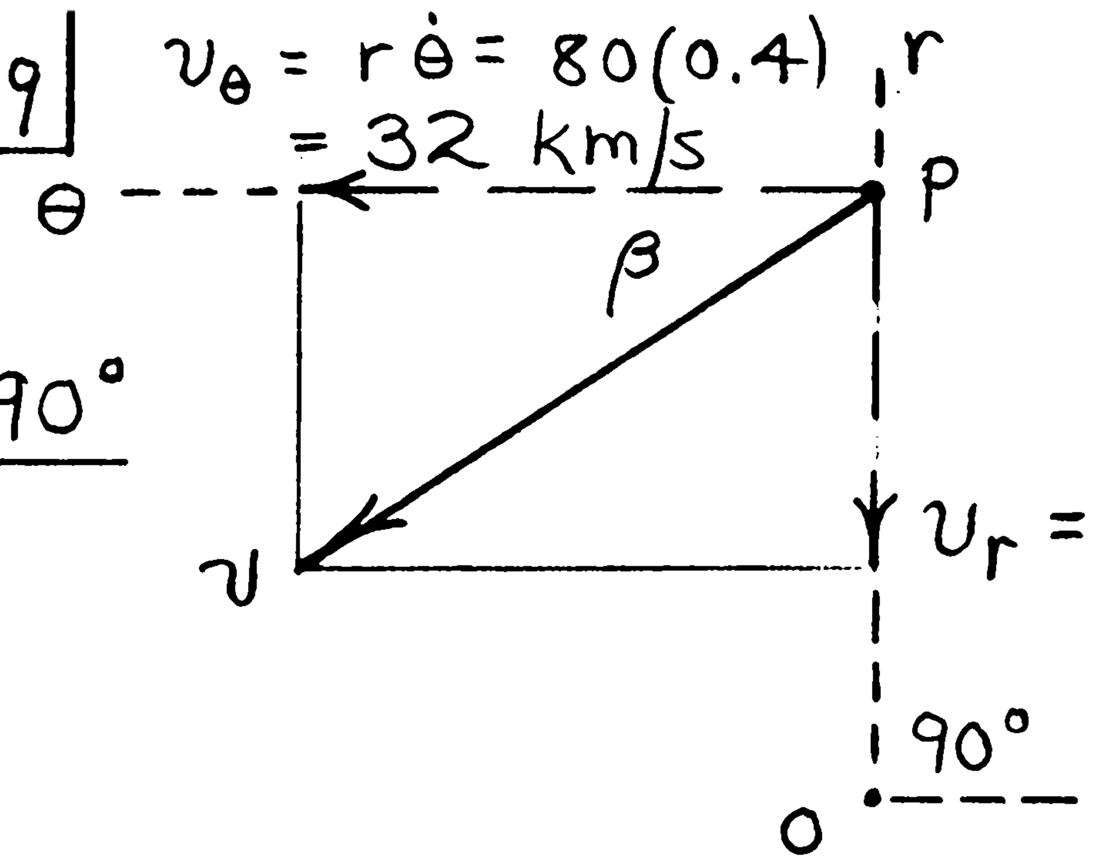
$\underline{\ddot{r} = -0.1345 \text{ m/s}^2}$

$a_\theta = a\frac{\sqrt{2}}{2} = r\ddot{\theta} + 2\dot{r}\dot{\theta} : 0.3\frac{\sqrt{2}}{2} = 2.32\ddot{\theta} + 2(0.424) \times (0.1830)$

$\underline{\ddot{\theta} = 0.025 \text{ rad/s}^2}$

**2/159**

$\theta = 90^\circ$

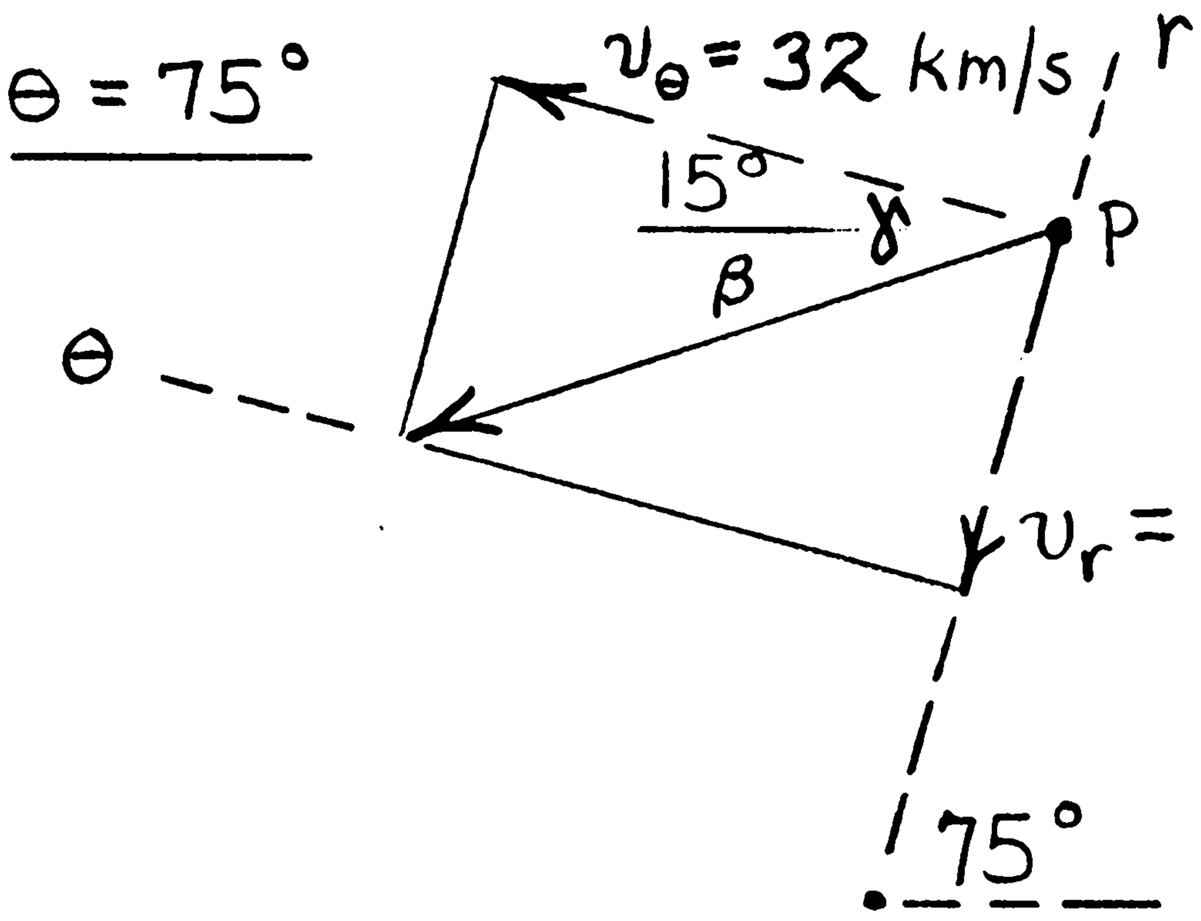



$r = \overline{OP} = 80$ km

$$v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = \underline{37.7 \text{ km/s}}$$

$$\beta = \tan^{-1}\frac{|v_r|}{v_\theta} = \tan^{-1}\frac{20}{32} = \underline{32.0^\circ}$$

$\theta = 75^\circ$



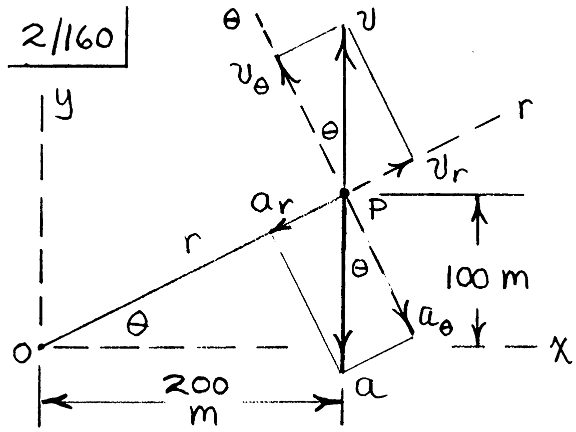

($v_\theta$ & $v_r$ unchanged)

$\gamma = 32.0^\circ$

$\underline{v = 37.7 \text{ km/s}}$ (unchanged)

$\beta = \gamma - 15^\circ = 32.0 - 15^\circ = \underline{17.01^\circ}$

2/160




$r = \sqrt{200^2 + 100^2} = \underline{224 \text{ m}}$

$\theta = \tan^{-1}\frac{100}{200} = \underline{26.6°}$

$v_r = \dot{r} = v \sin\theta = 15 \sin 26.6° = \underline{6.71 \text{ m/s}}$

$v_\theta = r\dot{\theta}: \quad 15 \cos 26.6° = 224\dot{\theta}, \quad \dot{\theta} = \underline{0.06 \text{ rad/s}}$

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06 \text{ m/s}^2$

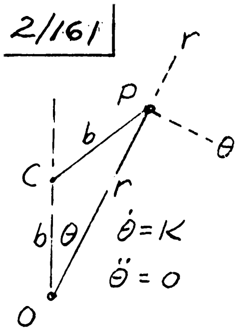
$a_r = \ddot{r} - r\dot{\theta}^2: \quad -12.06 \sin 26.6° = \ddot{r} - 224(0.06)^2$

$\underline{\ddot{r} = -4.59 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}: \quad -12.06 \cos 26.6° = 224\ddot{\theta} + 2(6.71)(0.06)$

$\underline{\ddot{\theta} = -0.0518 \text{ rad/s}^2}$

**2/161**




$r = 2b\cos\theta$, $\dot r = -2b\dot\theta\sin\theta = -2bK\sin\theta$

$\ddot r = -2bK^2\cos\theta$

$a_r = \ddot r - r\dot\theta^2 = -2bK^2\cos\theta - 2b\cos\theta\,(K^2)$

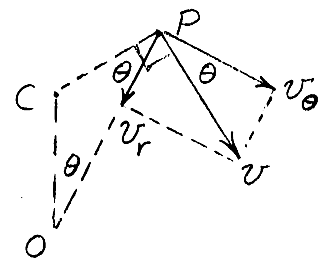
$\quad = -4bK^2\cos\theta$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(-2bK\sin\theta)K$

$\quad = -4bK^2\sin\theta$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4bK^2\sqrt{\sin^2\theta + \cos^2\theta} = \underline{4bK^2}$$

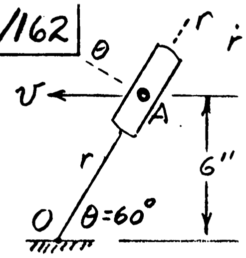
$v_r = \dot r = -2bK\sin\theta$, $v_\theta = r\dot\theta = 2bK\cos\theta$




$v = \sqrt{v_r^2 + v_\theta^2} = \underline{2bK \text{ const.}}$

(hence acceleration is directed from P to C and is $a_n = v^2/b$)

**2/162**




$\dot{r} = v_r = -v\cos\theta = -3\cos 60° = \underline{-1.5\ ft/sec}$

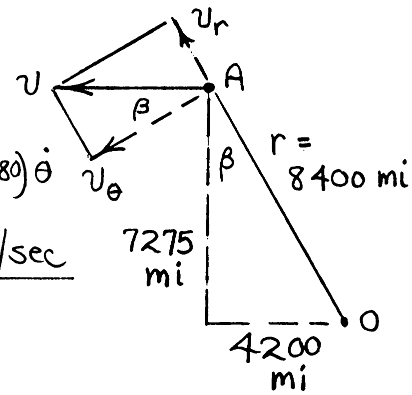
$r\dot{\theta} = v\sin\theta = 3\sin 60° = 2.60\ ft/sec$

$\dot{\theta} = \dfrac{2.60}{\frac{6}{12}/\sin 60°} = 2.6\sqrt{3} = \underline{4.50\ rad/sec}$

accel. $= 0$ so $a_r = \ddot{r} - r\dot{\theta}^2 = 0$, $\ddot{r} = r\dot{\theta}^2 = \dfrac{1}{\sqrt{3}}(4.50)^2 = \underline{11.69\ \dfrac{ft}{sec^2}}$

& $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$, $\ddot{\theta} = -\dfrac{2\dot{r}\dot{\theta}}{r} = -\dfrac{2(-1.5)(4.50)}{1/\sqrt{3}} = \underline{23.38\ \dfrac{rad}{sec^2}}$

2/163




$v = 12{,}149\left(\frac{5280}{3600}\right) = 17{,}819 \frac{\text{ft}}{\text{sec}}$

$v_\theta = r\dot\theta$ : $17{,}819 \cos 30° = 8400(5280)\dot\theta$

$\underline{\dot\theta = 3.48(10^{-4}) \text{ rad/sec}}$

$v_r = \dot r$ : $17{,}819 \sin 30° = \dot r$

$\underline{\dot r = 8910 \frac{\text{ft}}{\text{sec}}}$

$\beta = \tan^{-1}\left(\frac{4200}{7275}\right)$
$= 30°$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta$ : $0 = 8400(5280)\ddot\theta + 2(8910)(3.48)(10^{-4})$

$\underline{\ddot\theta = -1.398(10^{-7}) \text{ rad/sec}^2}$

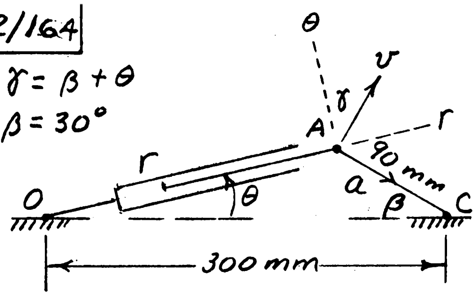
$a_r = \ddot r - r\dot\theta^2$ : $-7.159 = \ddot r - 8400(5280)(3.48 \times 10^{-4})^2$

$\underline{\ddot r = -1.790 \text{ ft/sec}^2}$

2/164

$\gamma = \beta + \theta$

$\beta = 30°$




$r\cos\theta + 90\cos 30° = 300$

$r\sin\theta = 90\sin 30°$

solve & get $r = 226.6$ mm

$\theta = 11.46°$

$\gamma = 30 + 11.46 = 41.46°$

$v = \overline{AC}\,\dot\beta = 90(60) = 5400$ mm/s or $5.40$ m/s

$a = a_n = \overline{AC}\,\dot\beta^2 = 90(60)^2 = 324(10^3)$ mm/s$^2$ or $324$ m/s$^2$

$a_t = \overline{AC}\,\ddot\beta = 0$

$v_r = \dot r = v\sin\gamma = 5.40\sin 41.46° = \underline{3.58 \text{ m/s}}$

$v_\theta = r\dot\theta = v\cos\gamma = 5.40\cos 41.46° = 4.05$ m/s, $\dot\theta = \dfrac{4.05}{0.2266} = \underline{17.86 \dfrac{\text{rad}}{\text{s}}}$

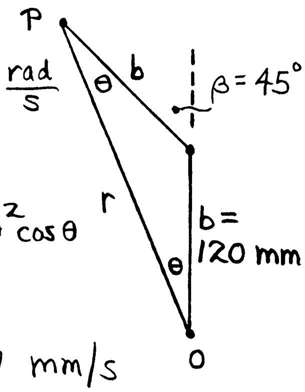
$a_r = \ddot r - r\dot\theta^2 = a\cos\gamma$, $\ddot r = 0.2266(17.86)^2 + 324\cos 41.46°$

$= 72.29 + 242.83 = \underline{315 \text{ m/s}^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = -a\sin\gamma$, $\ddot\theta = \dfrac{-2\dot r\dot\theta}{r} - \dfrac{a\sin\gamma}{r}$

$= -\dfrac{2(3.58)(17.86)}{0.2266} - \dfrac{324\sin 41.46°}{0.2266} = \underline{-1510 \dfrac{\text{rad}}{\text{s}^2}}$

▶**2/165** $2\theta = \beta = 45°$, $\theta = 22.5°$




Because $\theta = \frac{\beta}{2}$, $\dot{\theta} = \frac{\dot{\beta}}{2} = \frac{4}{2} = 2\ \frac{\text{rad}}{\text{s}}$

Also, $\ddot{\theta} = \frac{\ddot{\beta}}{2} = 0$

$r = 2b\cos\theta$, $\dot{r} = -2b\dot{\theta}\sin\theta$, $\ddot{r} = -2b\dot{\theta}^2\cos\theta$

$r = 2(120)\cos 22.5° = 222$ mm

$\dot{r} = -2(120)(2)\sin 22.5° = \underline{-183.7 \text{ mm/s}}$

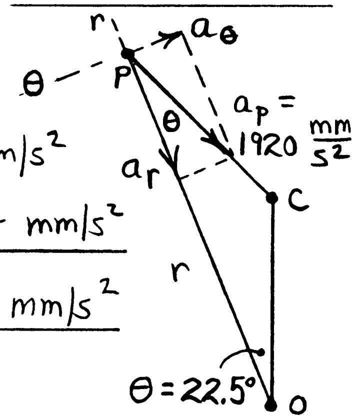
$\ddot{r} = -2(120)(2)^2\cos 22.5° = \underline{-887 \text{ mm/s}^2}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -887 - 222(2)^2 = -1774 \text{ mm/s}^2$ or $\underline{-1.774\ \frac{\text{m}}{\text{s}^2}}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 222(0) + 2(-183.7)(2) = -735 \text{ mm/s}^2$

or $\underline{a_\theta = -0.735 \text{ m/s}^2}$

Second Solution:



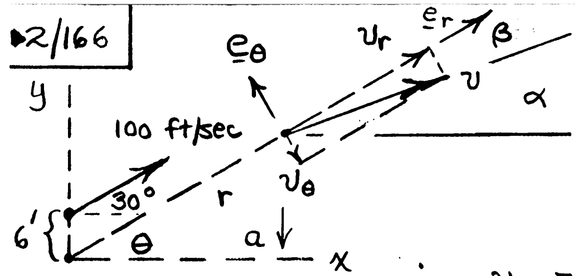

$(a_P)_t = b\ddot{\beta} = 0$

$(a_P)_n = b\dot{\beta}^2 = 120(4)^2 = 1920 \text{ mm/s}^2$

$a_r = -1920\cos 22.5° = \underline{-1774 \text{ mm/s}^2}$

$a_\theta = -1920\sin 22.5° = \underline{-735 \text{ mm/s}^2}$

▸2/166




$x = x_0 + v_{x_0} t$

$= 0 + 100 \cos 30° (0.5)$

$= 43.3$ ft

$\dot{x} = v_{x_0} = 100 \cos 30° = 86.6 \frac{\text{ft}}{\text{sec}}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30° (0.5) - 16.1 (0.5)^2 = 27.0$ ft

$\dot{y} = v_{y_0} - gt = 100 \sin 30° - 32.2(0.5) = 33.9$ ft/sec

$r = \sqrt{x^2 + y^2} = \sqrt{43.3^2 + 27.0^2} = \underline{51.0 \text{ ft}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(27.0/43.3) = \underline{31.9°}$

$\alpha = \tan^{-1}(v_y/v_x) = \tan^{-1}(33.9/86.6) = 21.4°$

$\beta = \theta - \alpha = 10.54°$, $v = \sqrt{\dot{x}^2 + \dot{y}^2} = 93.0$ ft/sec

$v_r = v \cos \beta = 93.0 \cos 10.54° = \underline{91.4 \text{ ft/sec} = \dot{r}}$

$v_\theta = -v \sin \beta = -93.0 \sin 10.54° = -17.02$ ft/sec

$-17.02 = r\dot{\theta} = 51.0\, \dot{\theta}$, $\underline{\dot{\theta} = -0.334 \text{ rad/sec}}$

$a = \ddot{y} = -32.2$ ft/sec$^2$

$a_r = -g \sin \theta = -32.2 \sin 31.9° = -17.03$ ft/sec$^2$

$-17.03 = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - 51.0(-0.334)^2$, $\underline{\ddot{r} = -11.35 \text{ ft/sec}^2}$

$a_\theta = -g \cos \theta = -32.2 \cos 31.9° = -27.3$ ft/sec$^2$

$-27.3 = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 51.0\ddot{\theta} + 2(91.4)(-0.334)$

$\underline{\ddot{\theta} = 0.660 \text{ rad/sec}^2}$

$\boxed{2/167}$ $\underline{v} = 4\underline{i} - 2\underline{j} - \underline{k}$ m/s, $v = \sqrt{4^2 + 2^2 + 1^2} = 4.58\ \frac{m}{s}$

$a_n = a \sin 20° = 8 \sin 20° = 2.74\ m/s^2$

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = \underline{7.67\ m}$

$\dot{v} = a_t = a \cos 20° = 8 \cos 20° = \underline{7.52\ m/s^2}$

2/168 $v_{z_0} = 500 \sin 60° = 433$ ft/sec

$v_{xy_0} = 500 \cos 60° = 250$ ft/sec

$v_z = v_{z_0} - gt = 433 - 32.2(20) = \underline{-211 \text{ ft/sec}}$

$v_{xy} = v_{xy_0} = \text{constant} = 250$ ft/sec

$v_x = v_{xy} \cos 20° = 250 \cos 20° = \underline{235 \text{ ft/sec}}$

$v_y = v_{xy} \sin 20° = 250 \sin 20° = \underline{85.5 \text{ ft/sec}}$

$d_{xy} = v_{xy} t = 250(20) = 5000$ ft

$x = d_{xy} \cos 20° = 5000 \cos 20° = \underline{4700 \text{ ft}}$

$y = d_{xy} \sin 20° = 5000 \sin 20° = \underline{1710 \text{ ft}}$

$z = v_{z_0} t - \frac{1}{2} g t^2 = 433(20) - 16.1(20)^2 = \underline{2220 \text{ ft}}$

$\underline{a_x = a_y = 0, \quad a_z = -g = -32.2 \text{ ft/sec}^2}$

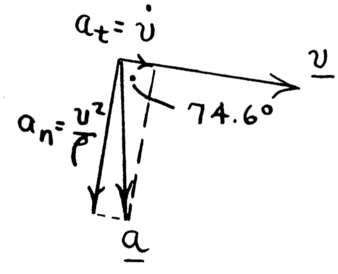
2/169 $\underline{v} = 6\underline{i} - 3\underline{j} + 2\underline{k}$ m/s, $v = \sqrt{6^2+3^2+2^2} = 7\frac{m}{s}$

$\underline{a} = 3\underline{i} - \underline{j} - 5\underline{k}$ m/s², $a = \sqrt{3^2+1^2+5^2} = \sqrt{35}\frac{m}{s^2}$

$$\theta = \cos^{-1}\left[\frac{\underline{v}\cdot\underline{a}}{va}\right] = \cos^{-1}\left[\frac{6(3)-3(-1)-2(5)}{7\sqrt{35}}\right]$$

$$= \underline{74.6^\circ}$$

In osculating plane:



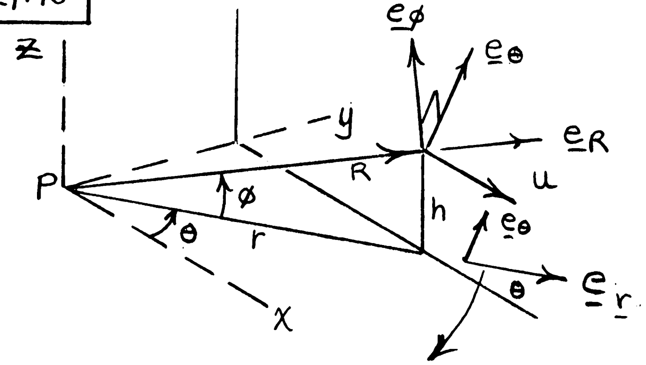

$\dot{v} = a\cos 74.6^\circ = \underline{1.571 \text{ m/s}^2}$

$a_n = a\sin 74.6^\circ = 5.70 \text{ m/s}^2$

$= \frac{v^2}{\rho}$

$\therefore \rho = \frac{v^2}{a_n} = \frac{7^2}{5.70} = \underline{8.59\text{ m}}$

2/170




Resolve $u$ along $e_\theta$ & $e_r$ directions:

$e_\theta$ : $\underline{-u \sin\theta} = v_\theta$

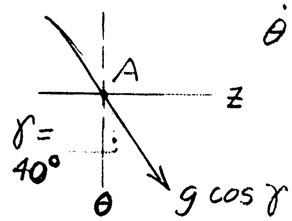
$e_r$ : $u\cos\theta = v_r$

Now resolve $v_r$ along $e_R$ & $e_\phi$ directions:

$e_R$ : $v_r \cos\phi = \underline{u\cos\theta\cos\phi} = v_R$

$e_\phi$ : $-v_r \sin\phi = \underline{-u\cos\theta\sin\phi} = v_\phi$

**2/171** $v_\theta = r\dot\theta$ & $v_\theta = v\cos\gamma$ so $\dot\theta = \dfrac{v}{r}\cos\gamma$




$$\dot\theta = \frac{15}{5}(0.7660) = 2.298 \text{ rad/s}$$

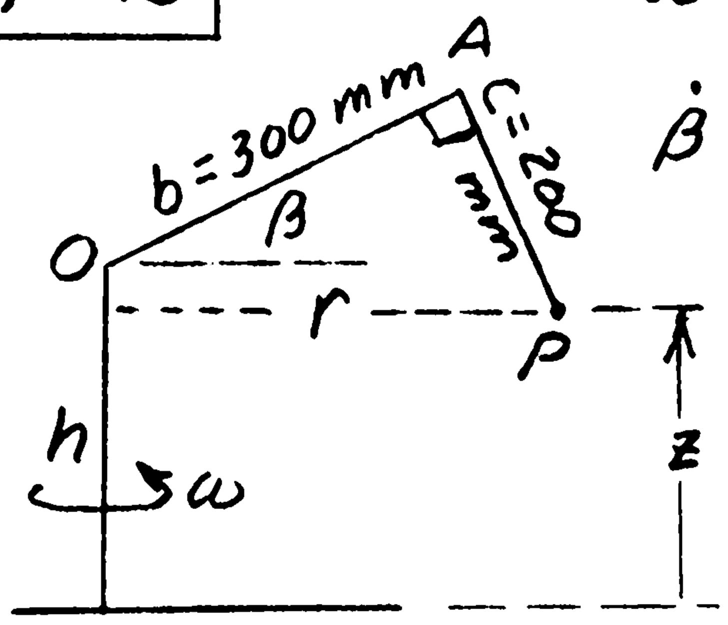
$$a_\theta = g\cos^2\gamma = 9.81(0.7660)^2 = 5.76 \text{ m/s}^2$$

$$a_z = g\cos\gamma\sin\gamma = 9.81(0.7660)(0.6428) = 4.83 \text{ m/s}^2$$

$$a_r = \ddot r - r\dot\theta^2 = 0 - 5(2.298)^2 = -26.41 \text{ m/s}^2$$

$$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = \underline{27.5 \text{ m/s}^2}$$

2/172




$\dot{\theta} = \omega = \frac{40}{180}\pi = 0.698$ rad/s

$\dot{\beta} = \frac{10}{180}\pi = 0.1745$ rad/s

$\ddot{\beta} = \frac{20}{180}\pi = 0.349$ rad/s$^2$

Use cylindrical coordinates

$r = b\cos\beta + c\sin\beta$, $\dot{r} = (-b\sin\beta + c\cos\beta)\dot{\beta}$

$\ddot{r} = (-b\cos\beta - c\sin\beta)\dot{\beta}^2 + (-b\sin\beta + c\cos\beta)\ddot{\beta}$

$z = h + b\sin\beta - c\cos\beta$, $\dot{z} = (b\cos\beta + c\sin\beta)\dot{\beta}$

$\ddot{z} = (-b\sin\beta + c\cos\beta)\dot{\beta}^2 + (b\cos\beta + c\sin\beta)\ddot{\beta}$

For $\beta = 30^\circ$, $\dot{r} = (-300\times0.5 + 200\times0.866)(0.1745) = 4.050$ mm/s

$\ddot{r} = (-300\times0.866 - 200\times0.5)(0.1745)^2$

$+(-300\times0.5 + 200\times0.866)(0.349) = -2.860$ mm/s$^2$

$\ddot{z} = (-300\times0.5 + 200\times0.866)(0.1745)^2$

$+(300\times0.866 + 200\times0.5)(0.349) = 126.30$ mm/s$^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -2.860 - 359.8(0.698)^2 = -178.23$ mm/s$^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.049)(0.698) = 5.65$ mm/s$^2$

$a_z = \ddot{z} = 126.30$ mm/s$^2$

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

$= \underline{218.5 \text{ mm/s}^2}$

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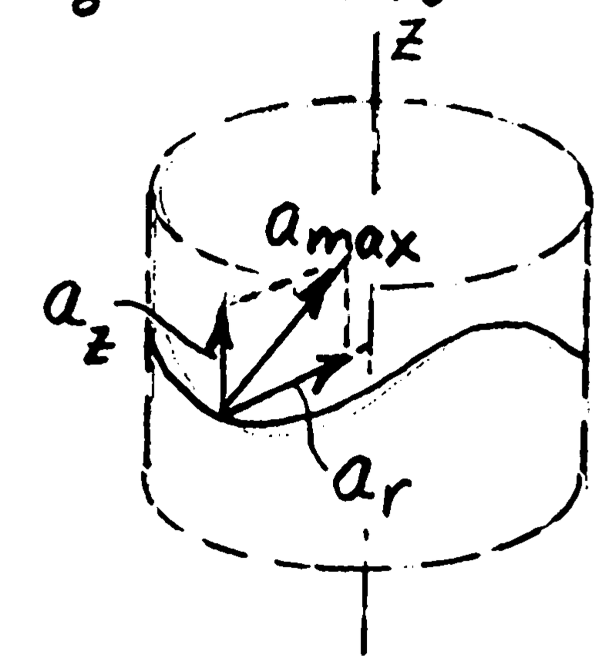
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\omega^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi n t) = -4n^2\pi^2 z_0 \sin 2\pi n t$




$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi n t)^2}$

$a_{max} = \underline{\sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}}$

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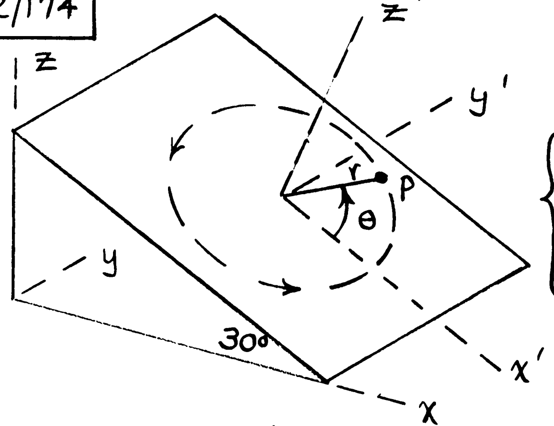


$\theta = \omega t = \frac{v}{r} t$

$\dot{\theta} = \frac{v}{r}$

$$\begin{cases} x' = r\cos\theta \\ y' = r\sin\theta \\ z' = 0 \end{cases}$$

$$\begin{cases} \dot{x}' = -r\dot{\theta}\sin\theta \\ \dot{y}' = r\dot{\theta}\cos\theta \\ \dot{z}' = 0 \end{cases}$$

$\ddot{x}' = -r\dot{\theta}^2\cos\theta$, $\ddot{y}' = -r\dot{\theta}^2\sin\theta$, $\ddot{z}' = 0$

Component relationships : (Similar for $\dot{x}$, $\ddot{x}$, etc.)

$$\begin{cases} x = x'\cos 30° + z'\sin 30° \\ y = y' \\ z = -x'\sin 30° + z'\cos 30° \end{cases}$$

Thus

$x = \frac{\sqrt{3}}{2} r\cos\frac{v}{r}t$, $y = r\sin\frac{v}{r}t$, $z = -\frac{1}{2} r\cos\frac{v}{r}t$

$\dot{x} = -\frac{\sqrt{3}}{2} v\sin\frac{v}{r}t$, $\dot{y} = v\cos\frac{v}{r}t$ $\dot{z} = \frac{1}{2} v\sin\frac{v}{r}t$

$\ddot{x} = -\frac{\sqrt{3}}{2}\frac{v^2}{r}\cos\frac{v}{r}t$, $\ddot{y} = -\frac{v^2}{r}\sin\frac{v}{r}t$, $\ddot{z} = \frac{1}{2}\frac{v^2}{r}\cos\frac{v}{r}t$

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Note that position relationships do not include the constants associated with the origin positions.

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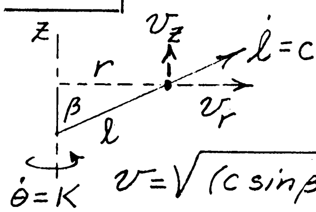



$$v_r = \dot{l}\sin\beta = c\sin\beta$$

$$v_\theta = r\dot{\theta} = (l\sin\beta)K = Kl\sin\beta$$

$$v_z = \dot{l}\cos\beta = c\cos\beta$$

$$v = \sqrt{(c\sin\beta)^2 + (Kl\sin\beta)^2 + (c\cos\beta)^2}$$

$$= \underline{\sqrt{c^2 + K^2 l^2 \sin^2\beta}}$$

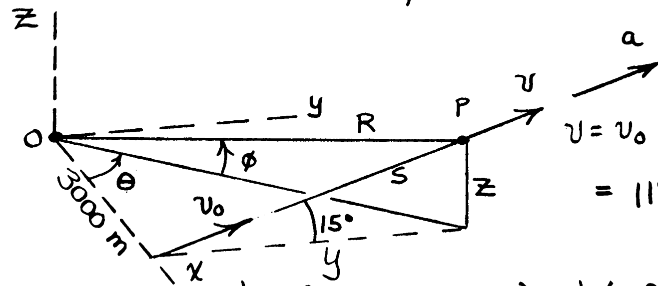
$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - l\sin\beta\,(K^2) = -K^2 l\sin\beta$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2c\sin\beta\,(K) = 2Kc\sin\beta$$

$$a_z = \dot{v}_z = 0$$

$$a = \sqrt{(-K^2 l\sin\beta)^2 + (2Kc\sin\beta)^2} = \underline{K\sin\beta\sqrt{K^2 l^2 + 4c^2}}$$

2/176 $a = 0.8\ \text{m/s}^2,\ v_0 = \dfrac{250}{3.6} = 69.4\ \text{m/s}^2$



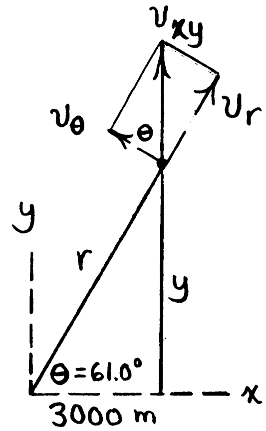

$v = v_0 + at = 69.4 + 0.8(60)$

$= 117.4\ \text{m/s}$

$s = v_0 t + \frac{1}{2}at^2 = 69.4(60) + \frac{1}{2}(0.8)(60)^2 = 5610\ \text{m}$

$y = 5610 \cos 15° = 5420\ \text{m}$

$\theta = \tan^{-1} \dfrac{5420}{3000} = 61.0°$




$r = \sqrt{3000^2 + 5420^2} = 6190\ \text{m}$

$v_{xy} = 117.4 \cos 15° = 113.4\ \text{m/s}$

$v_r = \dot{r} = 113.4 \sin 61.0° = \underline{99.2\ \text{m/s}}$

$v_\theta = r\dot{\theta} = 113.4 \cos 61.0°$

$\dot{\theta} = \dfrac{113.4 \cos 61.0°}{6190} = \underline{8.88\,(10^{-3})\ \text{rad/s}}$

$\dot{z} = v_z = v \sin 15° = 117.4 \sin 15° = \underline{30.4\ \text{m/s}}$

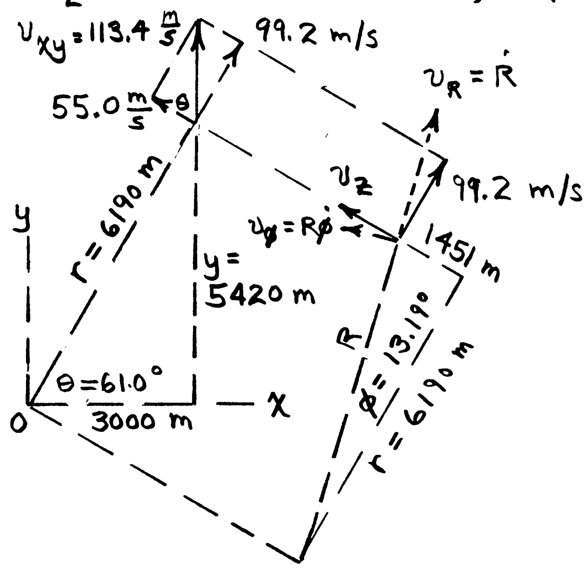
2/177 From the solution to Prob. 2/176,

$v = 117.4$ m/s, $\theta = 61.0°$, $y = 5420$ m, $r = 6190$ m

$z = y \tan 15° = 5420 \tan 15° = 1451$ m

$\phi = \tan^{-1}\frac{1451}{6190} = 13.19°$, $v_{xy} = 117.4 \cos 15° = 113.4$ m/s

$v_z = v \sin 15° = 30.4$ m/s, $R = \sqrt{1451^2 + 6190^2} = 6360$ m




$v_R = \dot{R} = 99.2 \cos 13.19° + 30.4 \sin 13.19° = \underline{103.6 \text{ m/s}}$

$v_\theta = R\dot{\theta}\cos\phi = r\dot{\theta}$, $\dot{\theta} = \frac{55.0}{6190} = \underline{8.88(10^{-3}) \text{ rad/s}}$

$v_\phi = R\dot{\phi} = 30.4 \cos 13.19° - 99.2 \sin 13.19° = 6.95$ m/s

$\dot{\phi} = \frac{6.95}{6360} = \underline{1.093\ (10^{-3})\ \text{rad/s}}$

2/178 $R = 0.75 + 0.5 = 1.25 \text{ m}$, $\dot{R} = 0.2 \text{ m/s}$, $\ddot{R} = -0.3 \frac{\text{m}}{\text{s}^2}$

$\phi = 30°$, $\dot{\phi} = 10\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\phi} = 0$, $\dot{\theta} = 20\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\theta} = 0$

$$\begin{cases} v_R = \dot{R} = 0.2 \text{ m/s} \\ v_\theta = R\dot{\theta}\cos\phi = 1.25\left(20\frac{\pi}{180}\right)\cos 30° = 0.378 \frac{\text{m}}{\text{s}} \\ v_\phi = R\dot{\phi} = 1.25\left(10\frac{\pi}{180}\right) = 0.218 \text{ m/s} \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{0.480 \text{ m/s}}$$

$$\begin{aligned} a_R &= \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi \\ &= -0.3 - 1.25\left(10\frac{\pi}{180}\right)^2 - 1.25\left(20\frac{\pi}{180}\right)^2\cos^2 30° \\ &= -0.4523 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} a_\theta &= \cos\phi\left[2\dot{R}\dot{\theta} + R\ddot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi \\ &= \cos 30°\left[2(0.2)\left(20\frac{\pi}{180}\right) + 1.25(0)\right] \\ &\quad - 2(1.25)\left(10\frac{\pi}{180}\right)\left(20\frac{\pi}{180}\right)\sin 30° = 0.0448 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$\begin{aligned} a_\phi &= 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi \\ &= 2(0.2)\left(10\frac{\pi}{180}\right) + 1.25(0) + 1.25\left(20\frac{\pi}{180}\right)^2 0.5\,\frac{\sqrt{3}}{2} \\ &= 0.1358 \text{ m/s}^2 \end{aligned}$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.474 \text{ m/s}^2}$$

2/179

$$R = (r^2+h^2)^{1/2}, \quad \dot{R} = \frac{1}{2}\frac{2r\dot{r}}{\sqrt{r^2+h^2}} = \frac{r\dot{r}}{\sqrt{r^2+h^2}}$$

$$r = 2b\sin\frac{\beta}{2}, \quad \dot{r} = b\dot{\beta}\cos\frac{\beta}{2}$$

$$\theta = \beta/2, \quad \dot{\theta} = \dot{\beta}/2$$

$$u = b\dot{\beta}, \quad \dot{\beta} = \text{constant}$$

$$v_R = \dot{R} = \frac{2b\sin\frac{\beta}{2}\,b\dot{\beta}\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}} = \frac{b^2\dot{\beta}\sin\beta}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}$$

$$= \underline{\frac{bu\sin\beta}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}}$$

$$v_\theta = R\dot{\theta}\cos\phi = r\dot{\theta} = 2b\sin\frac{\beta}{2}\frac{u}{2b} = \underline{u\sin\frac{\beta}{2}}$$

$$v_\phi = R\dot{\phi}, \quad \sin\phi = \frac{h}{R}, \quad \cos\phi\,\dot{\phi} = \frac{d}{dt}\left(\frac{h}{R}\right) = -\frac{h}{R^2}\dot{R}$$

$$\dot{\phi} = -\frac{h}{R^2}\dot{R}\frac{R}{r} = -\frac{h\dot{R}}{rR}$$

$$\text{So } v_\phi = \frac{-h\dot{R}}{r} = \frac{-hr\dot{r}}{\sqrt{r^2+h^2}}\frac{1}{r} = \frac{-hb\dot{\beta}\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}} = \underline{\frac{-hu\cos\frac{\beta}{2}}{\sqrt{4b^2\sin^2\frac{\beta}{2}+h^2}}}$$

2/180 Spherical coordinates

$$v_R = \dot{R} = 0.5 \text{ m/s}$$

$$v_\theta = R\dot{\theta}\cos\phi = 15\left(10\tfrac{\pi}{180}\right)\cos 30^\circ = 2.27 \text{ m/s}$$

$$v_\phi = R\dot{\phi} = 15\left(7\tfrac{\pi}{180}\right) = 1.833 \text{ m/s}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$

$$= 0 - 15\left(7\tfrac{\pi}{180}\right)^2 - 15\left(10\tfrac{\pi}{180}\right)^2\cos^2 30^\circ = -0.567 \text{ m/s}^2$$

$$a_\theta = \frac{\cos\phi}{R}\left[R^2\ddot{\theta} + 2R\dot{R}\dot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$$

$$= \frac{\cos 30^\circ}{15}\left[0 + 2(15)(0.5)\left(10\tfrac{\pi}{180}\right)\right] - 2(15)\left(10\tfrac{\pi}{180}\right)\left(7\tfrac{\pi}{180}\right)\sin 30^\circ$$

$$= -0.1687 \text{ m/s}^2$$

$$a_\phi = \frac{1}{R}\left[R^2\ddot{\phi} + 2\dot{R}\dot{\phi}\right] + R\dot{\theta}^2\sin\phi\cos\phi$$

$$= \frac{1}{15}\left[0 + 2(15)(0.5)\left(7\tfrac{\pi}{180}\right)\right] + 15\left(10\tfrac{\pi}{180}\right)^2\sin 30^\circ\cos 30^\circ$$

$$= 0.320 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$$

$\boxed{2/181}$ $R = 24\ m$ const, $\dot{\theta} = \omega = \frac{2(2\pi)}{60} = \frac{\pi}{15}$ rad/s, $\ddot{\theta} = 0$

$\beta = 30°$, $\varphi = \frac{\pi}{2} - \beta$, $\dot{\varphi} = -\dot{\beta} = -0.10$ rad/s, $\ddot{\varphi} = -\ddot{\beta} = 0$

$v_R = \dot{R} = 0$, $v_\theta = R\dot{\theta}\cos\varphi = 24\left(\frac{\pi}{15}\right)\frac{1}{2} = 2.51$ m/s

$v_\varphi = R\dot{\varphi} = 24(-0.10) = -2.4$ m/s

$v = \sqrt{(2.51)^2 + (2.4)^2} = \underline{3.48\ m/s}$

$a_R = \ddot{R} - R\dot{\varphi}^2 - R\dot{\theta}^2\cos^2\varphi = 0 - 24(-0.10)^2 - 24\left(\frac{\pi}{15}\right)^2\left(\frac{1}{2}\right)^2 = -0.503\ \frac{m}{s^2}$

$a_\theta = \frac{\cos\varphi}{R}\frac{d}{dt}(R^2\dot{\theta}) - 2R\dot{\theta}\dot{\varphi}\sin\varphi = 0 - 2(24)\frac{\pi}{15}(-0.10)\frac{\sqrt{3}}{2} = 0.871\ \frac{m}{s^2}$

$a_\varphi = \frac{1}{R}\frac{d}{dt}(R^2\dot{\varphi}) + R\dot{\theta}^2\sin\varphi\cos\varphi = 0 + 24\left(\frac{\pi}{15}\right)\frac{\sqrt{3}}{2}\frac{1}{2} = 0.456\ m/s^2$

$a = \sqrt{(-0.503)^2 + (0.871)^2 + (0.456)^2} = \underline{1.104\ m/s^2}$

**2/182** Use Eq. 2/19 where $\dot{\varphi} = -\dot{\beta}$, $R = L$, $\dot{\theta} = \omega$

$$a_R = 0 - 1.2\left(-\frac{3}{2}\right)^2 - 1.2(2)^2\frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$$

$$a_\theta = \frac{\sin\beta}{L}(2L\dot{L}\,\omega + 0) + 2L\omega\dot{\beta}\cos\beta = 2\omega(\dot{L}\sin\beta + L\dot{\beta}\cos\beta)$$

$$= 2(2)\left(0.9\frac{1}{\sqrt{2}} + 1.2\left(\frac{3}{2}\right)\frac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2}$$

$$a_\varphi = -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\frac{3}{2} + 1.2(2^2)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$

$$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$$

▶2/183 $R = \text{const}$ $\theta = \omega t$ $\sin\phi = z/R$

$z = \frac{h}{2}(1-\cos 2\theta)$, $\dot{z} = \omega h \sin 2\theta$ where $\dot{\theta} = \omega$

$(\cos\phi)\dot{\phi} = \frac{1}{R}\dot{z}$, $\dot{\phi} = \frac{\omega h \sin 2\theta}{R\cos\phi}$

$v_R = \dot{R} = 0$

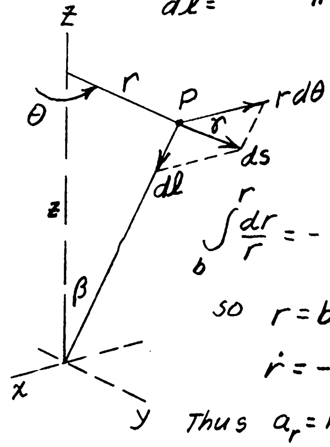
$v_\theta = R\dot{\theta}\cos\phi = R\omega\sqrt{1-\sin^2\phi} = R\omega\sqrt{1-\left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}$

$v_\phi = R\dot{\phi} = \frac{\omega h \sin 2\theta}{\cos\phi} = h\omega\frac{\sin 2\theta}{\sqrt{1-\left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}}$

When $\theta = \omega t = \pi/4$, $1-\cos 2\theta = 1$ so that

$\underline{v_\theta = R\omega\sqrt{1-(h/2R)^2}}$, $\underline{v_\phi = \frac{h\omega}{\sqrt{1-(h/2R)^2}}}$, $\underline{v_R = 0}$

▶2/184 $ds$ = differential distance along curve
$d\ell$ = " " in direction of cone element




$r = \frac{b}{h} z$, $\tan\beta = b/h$

$d\ell = r\,d\theta \tan\gamma$

$dr = -d\ell \sin\beta = -r\,d\theta \tan\gamma \sin\beta$

$$\int_b^r \frac{dr}{r} = -\tan\gamma \sin\beta \int_0^\theta d\theta\,, \quad \ln r\Big]_b^r = -\tan\gamma\sin\beta\;\theta$$

so $r = be^{-\tan\gamma \sin\beta\,\theta}$, $r = be^{-K\theta}$ where $K = \tan\gamma\sin\beta$

$\dot r = -bK\dot\theta e^{-K\theta}$, $\ddot r = bK^2\dot\theta^2 e^{-K\theta}$, $\ddot\theta = 0$

Thus $a_r = \ddot r - r\dot\theta^2 = b\dot\theta^2 e^{-K\theta}(K^2 - 1)$

or $\underline{a_r = b\dot\theta^2(\tan^2\gamma \sin^2\beta - 1)\, e^{-\theta\tan\gamma\sin\beta}}$

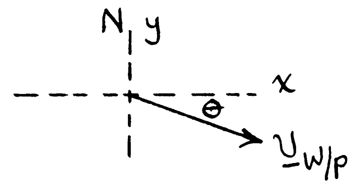
$\underline{\text{where } \beta = \tan^{-1}\frac{b}{h}}$

2/185 (a) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P$

$$= 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - (-4\underline{i}) = \underline{6.12\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$$

or $v_{W/P} = (6.12^2 + 2.12^2)^{1/2} = \underline{6.48\ \text{mi/hr}}$

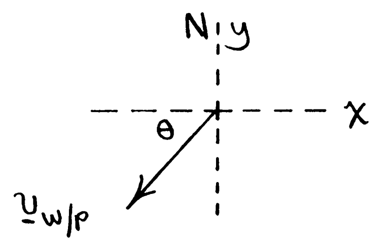
at $\theta = \tan^{-1}\dfrac{2.12}{6.12} = \underline{19.11^\circ \text{ south of east}}$




(b) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - 4\underline{i} = \underline{-1.879\underline{i} - 2.12\underline{j}\ \frac{\text{mi}}{\text{hr}}}$

or $v_{W/P} = (1.879^2 + 2.12^2)^{1/2} = \underline{2.83\ \text{mi/hr}}$

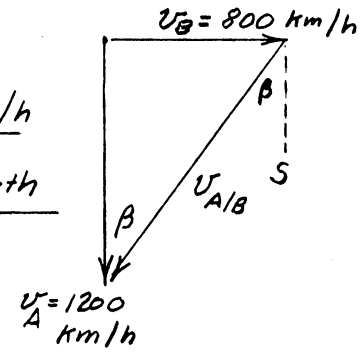
at $\theta = \tan^{-1}\dfrac{2.12}{1.879} = \underline{48.5^\circ \text{ south of west}}$



2/186 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_{A/B} = \sqrt{(1200)^2 + (800)^2} = \underline{1442 \text{ km/h}}$

$\beta = \tan^{-1}\dfrac{800}{1200} = \underline{33.7° \text{ west of south}}$



2/187 $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$

$= 120[\cos 15°\underline{i} + \sin 15°\underline{j}] - 90[\cos 60°\underline{i} + \sin 60°\underline{j}]$

$= \underline{70.9\underline{i} - 46.9\underline{j} \quad \text{km/h}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \underline{0} - 3(-\cos 60°\underline{i} - \sin 60°\underline{j})$

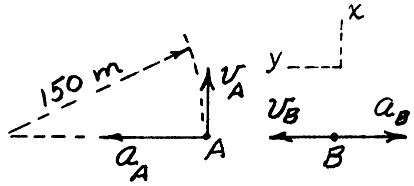
$= \underline{1.5\underline{i} + 2.60\underline{j} \quad \text{m/s}^2}$

$y$

$x$

$\underline{2/188}$ If $\frac{d}{dt}(\overline{AB}) = |v_{A/B}|$ then $\underline{v}_{A/B}$ does not change direction, which requires that $\underline{|\underline{v}_A| / |\underline{v}_B| = \text{const}}$

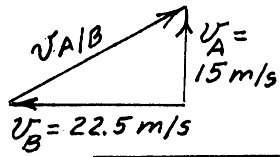
**2/189** $v_A = 54/3.6 = 15\ m/s$ , $v_B = 81/3.6 = 22.5\ m/s$




$$a_A = \frac{v_A^2}{\rho} = \frac{15^2}{150} = 1.5\ m/s^2$$

$$a_B = 3\ m/s^2$$

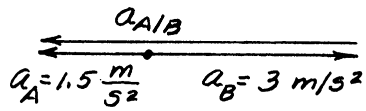
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$




$$v_{A/B} = \sqrt{(22.5)^2 + (15)^2}$$
$$= 27.0\ m/s$$

$$\underline{\underline{v}_{A/B} = 15\underline{i} - 22.5\underline{j}\ m/s}$$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

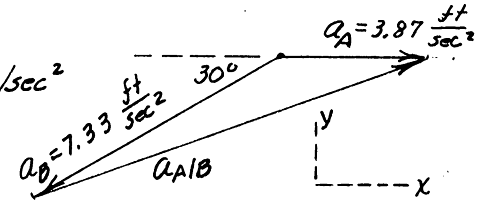



$$\underline{\underline{a}_{A/B} = 4.5\underline{j}\ m/s^2}$$

2/190 $\underset{-}{a}_A = \underset{-}{a}_B + \underset{-}{a}_{A/B}$

$a_A = v^2/\rho = (44)^2/500 = 3.87 \text{ ft/sec}^2$

$a_B = \frac{5}{30} 44 = 7.33 \text{ ft/sec}^2$




$\underset{-}{a}_{A/B} = (7.33 \cos 30° + 3.87)\underset{-}{i} + 7.33 \sin 30° \underset{-}{j}$

$= \underline{10.22\underset{-}{i} + 3.67\underset{-}{j}} \quad \text{ft/sec}^2$

2/191

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$ , $\Omega = 3(2\pi/60) = 0.314 \frac{\text{rad}}{\text{s}}$

$= \frac{18}{3.6}\underline{i} - 9(0.314)(\cos 45°\underline{i} - \sin 45°\underline{j})$

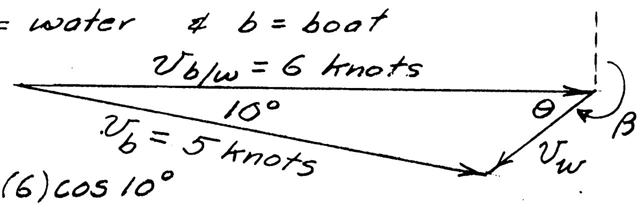
$= \underline{3.00\underline{i} + 2.00\underline{j} \text{ m/s}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = 3\underline{i} - 9(0.314)^2(-\cos 45°\underline{i} - \sin 45°\underline{j})$

$= \underline{3.63\underline{i} + 0.628\underline{j} \text{ m/s}^2}$

2/192 Let $w$ = water & $b$ = boat

$\underline{v}_b = \underline{v}_w + \underline{v}_{b/w}$



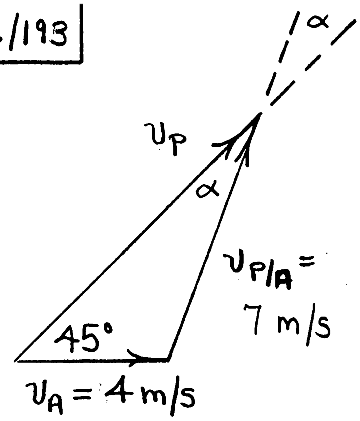

$v_w^2 = 5^2 + 6^2 - 2(5)(6)\cos 10°$

$= 1.91$, $\underline{v_w = 1.38 \text{ knots}}$

$5/\sin\theta = 1.38/\sin 10°$, $\sin\theta = 0.6280$, $\theta = 38.9°$

$\beta = 270 - 38.9 = \underline{231°}$

2/193




With P being the puck:

$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

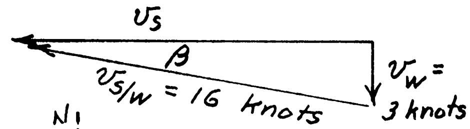
Law of sines :

$$\frac{\sin 45^\circ}{7} = \frac{\sin \alpha}{4}$$

$$\underline{\alpha = 23.8^\circ}$$

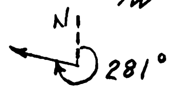
2/194 | $w$ = water, $s$ = ship

$\underline{v}_s = \underline{v}_w + \underline{v}_{s/w}$




$\beta = \sin^{-1} 3/16 = 10.8°$

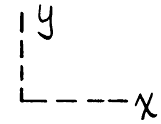
Heading $= 270 + 10.8 = \underline{281°}$




time $t = \dfrac{\text{Dist.}}{\text{vel.}} = \dfrac{24}{\sqrt{(16)^2 - 3^2}} = \underline{1.527 \text{ hr}}$

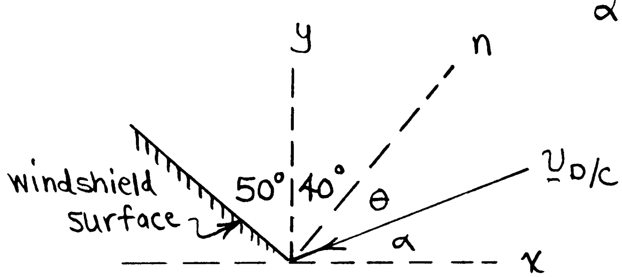
**2/195** Drop: $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85$ m/s

Car: $v_C = 100/3.6 = 27.8$ m/s




$\underline{v}_{D/C} = \underline{v}_D - \underline{v}_C = -10.85\underline{j} - 27.8\underline{i}$ m/s

$$\alpha = \tan^{-1}\frac{10.85}{27.8} = 21.3^\circ$$



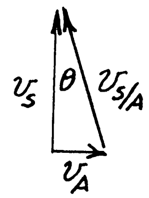

$40^\circ + \theta + \alpha = 90^\circ \Rightarrow$ $\underline{\theta = 28.7^\circ \text{ below normal}}$

2/196 | Let S = satellite , A = observer

$\underline{v}_S = \underline{v}_A + \underline{v}_{S/A}$ , $v_A = R\omega$

$= 6378(0.729)(10^{-4})(3600)$

$= 1674$ km/h (East)




$v_S = 27\,940$ km/h (North)

$\theta = \tan^{-1}\dfrac{1674}{27940} = 3.43°$

Satellite appears to travel <u>3.43° west of north</u>

2/197 Use $g = g_0 \left(\frac{R}{R+h}\right)^2$

For A, $g_A = 32.23 \left(\frac{3959}{3959+200}\right)^2 = 29.2 \text{ ft/sec}^2$

For B, $g_B = 32.23 \left(\frac{3959}{3959+22{,}300}\right)^2 = 0.733 \text{ ft/sec}^2$

$$\underset{\sim}{a}_{B/A} = \underset{\sim}{a}_B - \underset{\sim}{a}_A = +0.733\underset{\sim}{i} - (-29.2)\underset{\sim}{j}$$

$$= \underline{+0.733\underset{\sim}{i} + 29.2\underset{\sim}{j} \quad \text{ft/sec}^2}$$

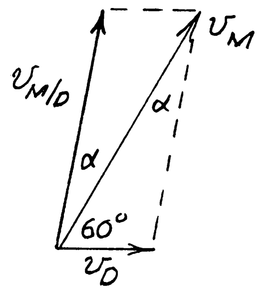
2/198 Let $v_D$ = velocity of destroyer $= 30(1.852)$
$= 55.6$ km/h

or $v_D = \dfrac{55.6}{3.6} = 15.43$ m/s

Let $v_M$ = horiz. velocity of missile

$(v_{M/D})$ = " " " " rel. to ship

$\underline{v}_M = \underline{v}_D + \underline{v}_{M/D}$ $= 75 \cos 30° = 65.0$ m/s

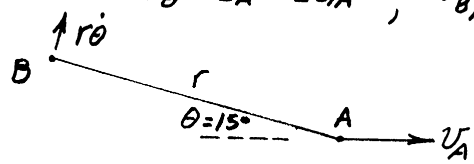


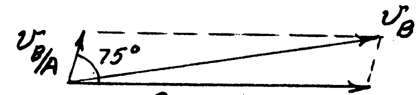

Law of sines,

$$\sin\alpha = \frac{v_D}{v_{M/D}} \sin 60° = \frac{15.43}{65.0}\frac{\sqrt{3}}{2} = 0.2058$$

$\underline{\alpha = 11.88°}$

**2/199** $\quad \underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$ , $\quad v_{B/A} = r\dot{\theta} = 60\left(\frac{5}{180}\pi\right) = 5.24 \text{ m/s}$




$v_A = \frac{200}{3.6} = 55.6 \text{ m/s}$

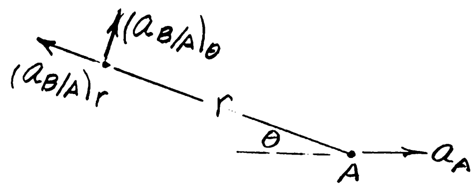
$$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264 \text{ (m/s)}^2$$

$v_B = 57.1$ m/s or $v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ , $a_A = 0$, $a_{B/A} = r\dot{\theta}^2 = 60\left(\frac{5\pi}{180}\right)^2 = 0.457 \text{ m/s}^2$

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

**2/200** $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

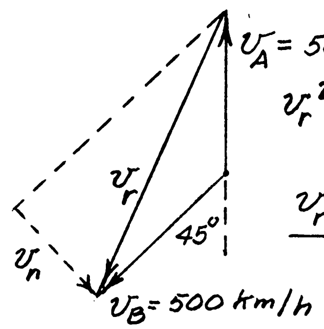



$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0 = 0$

$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 0 = 0$

Thus $a_B = a_A = \frac{5}{3.6} = \underline{1.389 \text{ m/s}^2}$

2/201 $\underline{v}_B = \underline{v}_A + \underline{v}_r$ where $\underline{v}_r = \underline{v}_{B/A}$




$v_r^2 = (500)^2 + (500)^2 + 2(500)(500)\cos 45°$

$= 854(10^3)$ $(km/h)^2$

$\underline{v_r = 924 \text{ km/h}}$

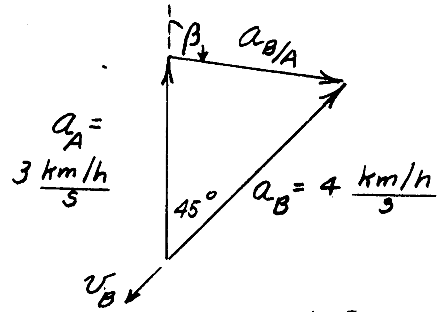
$v_n = 500 \cos 45° = \underline{354 \text{ km/h}}$

Let $h$ = possible constant difference in altitude

$\underline{k}$ = unit vector in vertical direction

Since $\frac{d}{dt}(h\underline{k}) = 0$, vertical separation has no influence on relative-velocity equations.

**2/202** $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$




$(a_{B/A})^2 = 3^2 + 4^2 - 2(3)(4)\cos 45°$
$= 8.03\ (\frac{km/h}{s})^2$

$a_{B/A} = 2.83\ \frac{km/h}{s}$

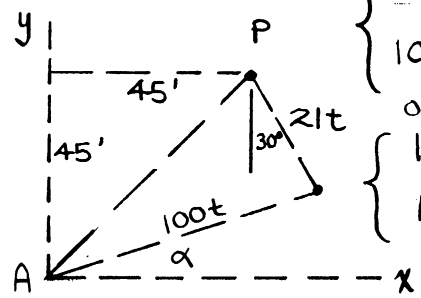
$= \frac{2.83}{3.6} = \underline{0.787\ m/s^2}$

$4^2 = 3^2 + (2.83)^2 + 2(3)(2.83)\cos\beta$ , $\cos\beta = \frac{-1.030}{17.00}$

$= -0.0606$

$\underline{\beta = 93.5°}$

2/203




$$\begin{cases} 100t\cos\alpha = 45 + 21t\sin 30° \\ 100t\sin\alpha = 45 - 21t\cos 30° \end{cases}$$

or

$$\begin{cases} 100\cos\alpha = \frac{45}{t} + 21\sin 30° \\ 100\sin\alpha = \frac{45}{t} - 21\cos 30° \end{cases}$$

Subtract $2^{\text{nd}}$ from $1^{\text{st}}$:

$$100(\cos\alpha - \sin\alpha) = 21\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) \text{ or } \cos\alpha - \sin\alpha = 0.287$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = 0.287 + \sin\alpha$$

SQBS & rearrange : $2\sin^2\alpha + 0.574\sin\alpha - 0.918 = 0$

Positive solution : $\underline{\alpha = 33.3°}$ ; Then $t = 0.616$ sec

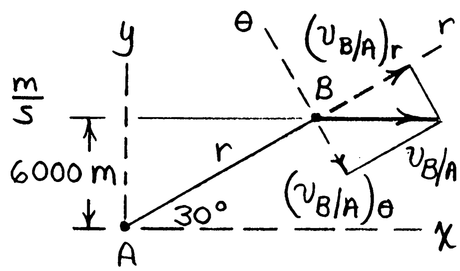
$$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 100[\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j}]$$

$$- 21[\sin 30°\,\underline{i} - \cos 30°\,\underline{j}]$$

$$= \underline{73.1\,\underline{i} + 73.1\,\underline{j} \text{ ft/sec}}$$

▶2/204 $v_{B/A} = v_B - v_A$

$= (1500-1000)/3.6 = 138.9 \ \frac{m}{s}$




$(v_{B/A})_r = \dot{r} = 138.9 \cos 30°$

$= 120.3 \ m/s$

$(v_{B/A})_\theta = r\dot{\theta} : -138.9 \sin 30° = \frac{6000}{\sin 30°} \dot{\theta}$

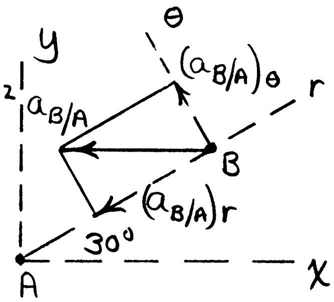
$\dot{\theta} = -0.00579 \ rad/s$

$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2 \ m/s^2$

$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$

$-1.2 \cos 30° = \ddot{r} - 12000(-0.00579)^2$

$\underline{\ddot{r} = -0.637 \ m/s^2}$




$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$1.2 \sin 30° = 12000\ddot{\theta} + 2(120.3)(-0.00579)$

$\underline{\ddot{\theta} = 0.1660(10^{-3}) \ rad/s^2}$

▶ 2/205 Find flight time $t$ :

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ : $7 = 3 + 100 \sin 30^\circ t - 16.1 t^2$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

Range $R = x_0 + v_{x_0} t = 0 + 100 \cos 30^\circ (3.02)$
$= 262$ ft

Fielder must run $262 - 220 = 41.8$ ft
in $(3.02 - 0.25)$ sec $\Rightarrow v_B = \frac{41.8}{2.77} = 15.08$ ft/sec

Velocity components of ball when caught:

$v_x = v_{x_0} = 100 \cos 30^\circ = 86.6$ ft/sec

$v_y = v_{y_0} - gt = 100 \sin 30^\circ - 32.2(3.02) = -47.4 \frac{\text{ft}}{\text{sec}}$

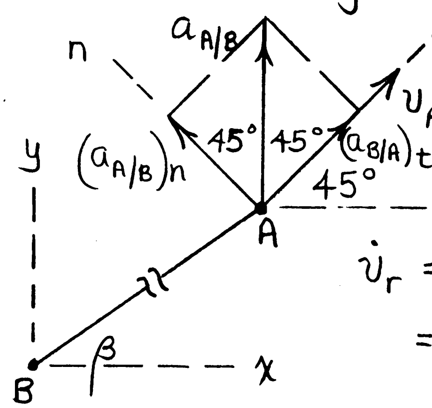
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6 \underline{i} - 47.4 \underline{j}) - 15.08 \underline{i}$

$= \underline{71.5 \underline{i} - 47.4 \underline{j} \text{ ft/sec}}$

▶2/206 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\underline{i} - (-50\underline{j}) = \underline{50\underline{i} + 50\underline{j}\ \text{m/s}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \dfrac{v_A^2}{\rho_A}\underline{j} - \underline{0} = \dfrac{50^2}{2000}\underline{j} = \underline{1.250\underline{j}\ \text{m/s}^2}$

(b) Use the results of part (a) for a normal-tangential analysis:




$v_{A/B} = \sqrt{50^2 + 50^2} = 70.7$ m/s

$\dot{v}_r = (a_{B/A})_t = a_{A/B}\cos 45°$

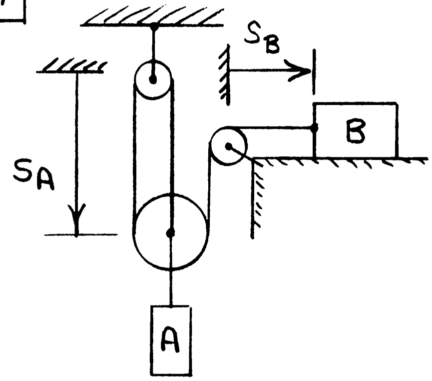
$= 1.250\,\dfrac{\sqrt{2}}{2} = \underline{0.884\ \text{m/s}^2}$

$(a_{B/A})_n = a_{B/A}\sin 45° = \dfrac{v_{A/B}^2}{\rho_r}$

$1.250\,\dfrac{\sqrt{2}}{2} = \dfrac{70.7^2}{\rho_r}$, $\underline{\rho_r = 5660\ \text{m}}$

(in n-direction from A)

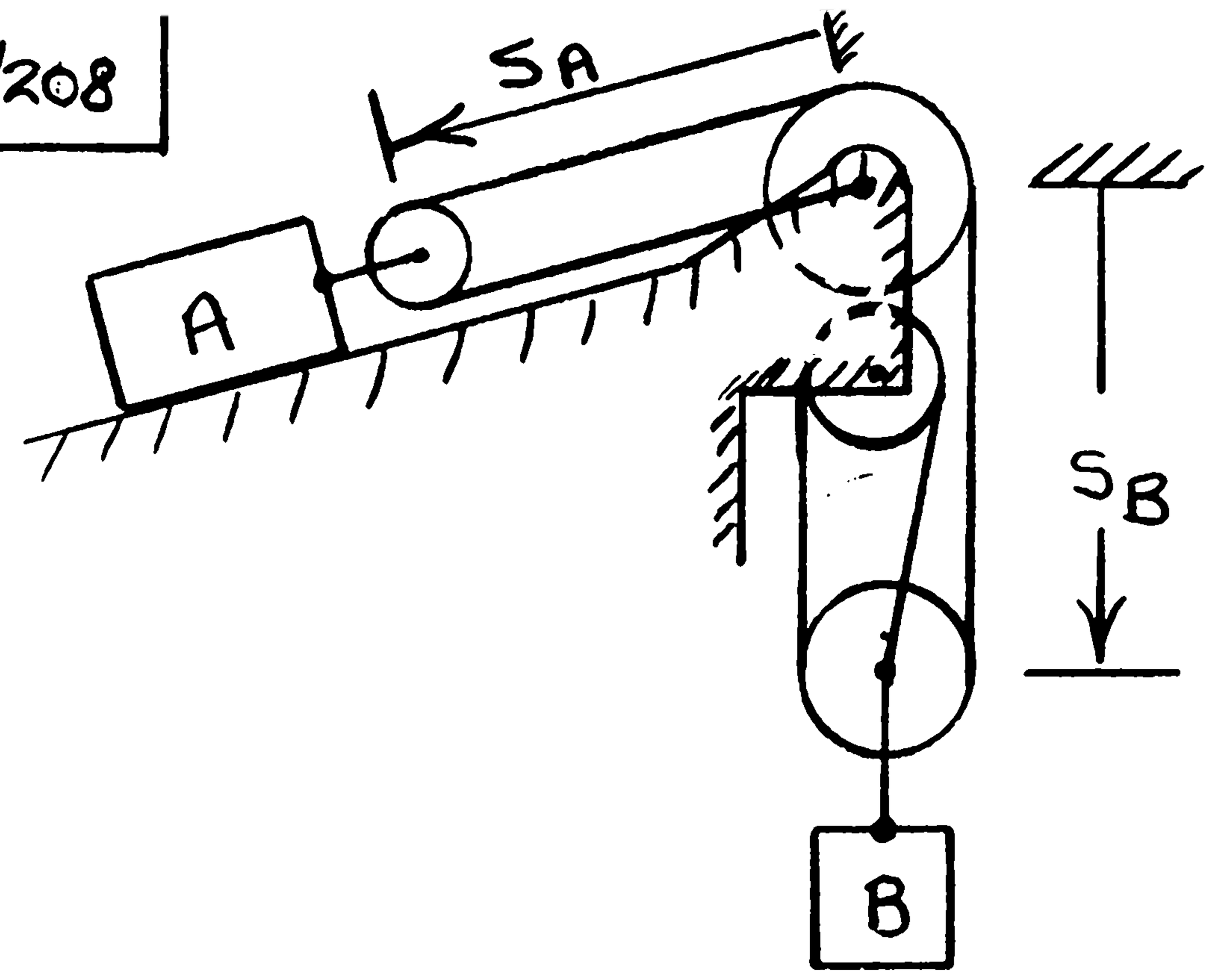
2/207




Length of cable $L = s_B + 3s_A$ + constants

$0 = v_B + 3v_A$, $v_A = -\frac{v_B}{3} = -\left(\frac{-1.2}{3}\right) = \underline{0.4 \frac{m}{s}}$ (down)

2/208




Cable length $L = 2s_A + 3s_B + \text{constants}$

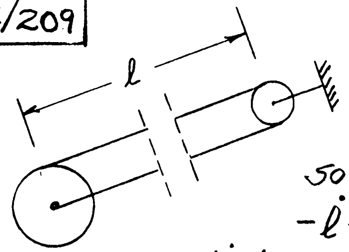
$0 = 2v_A + 3v_B$ , $0 = 2a_A + 3a_B$

$v_A = -\frac{3}{2} v_B = -\frac{3}{2}(2) = \underline{-3 \text{ ft/sec}}$

$a_A = -\frac{3}{2} a_B = -\frac{3}{2}(-0.5) = \underline{+0.75 \text{ ft/sec}^2}$

or $\begin{cases} v_A = 3 \text{ ft/sec up the incline} \\ \underline{a_A = 0.75 \text{ ft/sec}^2 \text{ down the incline}} \end{cases}$

**2/209**




Length of cable is

$$L = 2\ell$$

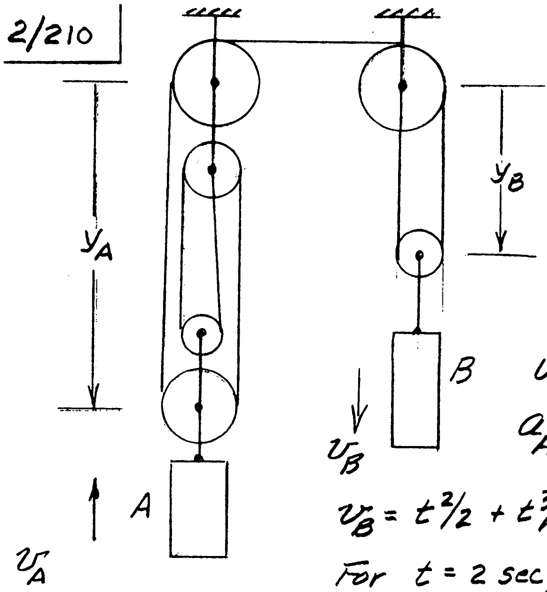
$$\dot{L} = 2\dot{\ell}$$

so velocity of truck is

$$-\dot{\ell} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20 \text{ mm/s}$$

$$\text{time } t = \frac{\text{distance}}{\text{velocity}} = \frac{4(10^3)}{20} = 200 \text{ s} \quad \text{or} \quad \underline{3 \text{ min } 20 \text{ s}}$$

2/210




Total length of cable to within constants is

$$L = 4y_A + 2y_B$$

$$0 = 4\dot{y}_A + 2\dot{y}_B$$

$$0 = 4\ddot{y}_A + 2\ddot{y}_B$$

Upward accel. of A is

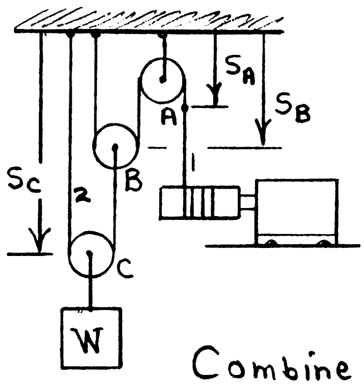
$$a_A = -\ddot{y}_A = \frac{1}{2}\ddot{y}_B = \frac{1}{2}a_B$$

$v_B = t^2/2 + t^3/6, \quad a_B = \dot{v}_B = t + t^2/2$

For $t = 2$ sec, $a_B = 2 + 4/2 = 4$ ft/sec$^2$

Thus $a_A = \frac{1}{2}a_B = \underline{2 \text{ ft/sec}^2}$

2/211




Let A be a point on cable 1:

$$L_1 = s_A + 2s_B$$

$$0 = v_A + 2v_B \qquad (1)$$

$$L_2 = s_C + (s_C - s_B)$$

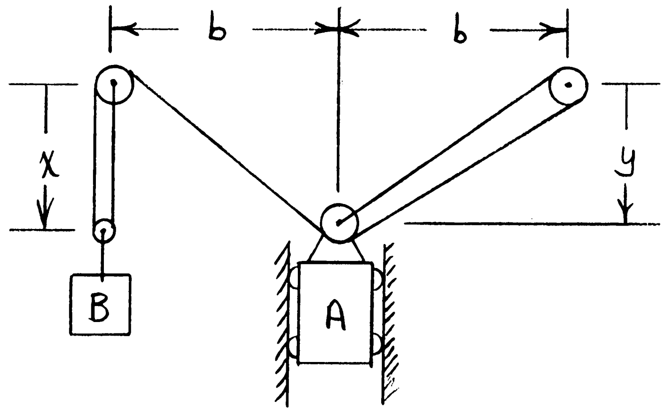
$$0 = 2v_C - v_B \qquad (2)$$

Combine (1) & (2) to obtain

$$v_C = -\frac{1}{4}v_A = -\frac{1}{4}(320) = -80 \text{ mm/s}$$

So G (and W) rises $h = 80(5) = \underline{400 \text{ mm}}$

2/212




The total length of the cable is

$$L = 2x + 3\sqrt{y^2+b^2} + \text{constant}$$

Differentiate to obtain

$$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2+b^2}}$$

With $\dot{x} = v_B$ & $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2+b^2}}v_A$$

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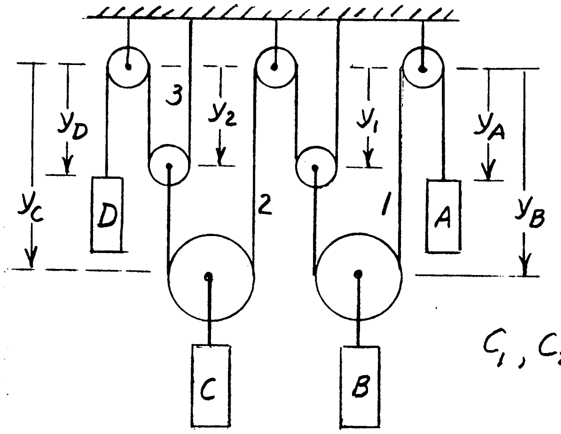



3 degrees of freedom

$C_1, C_2, C_3$ are constant lengths

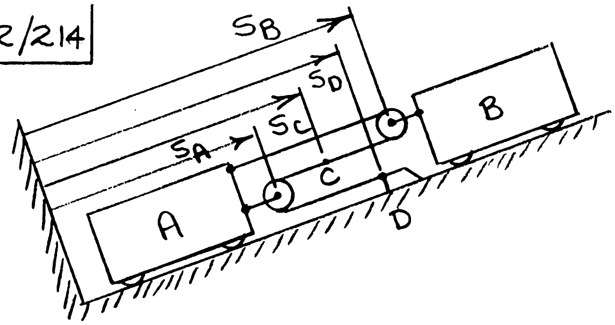
$L_1 = y_B + y_A + (y_B - y_1) + C_1$; $\quad 0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$

$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2 \quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$

$L_3 = 2y_2 + y_D + C_3, \quad 0 = 2\dot{y}_2 + \dot{y}_D$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & get $\underline{4v_A + 8v_B + 4v_C + v_D = 0}$

2/214




The cable length is $L = 2(s_B - s_A) + s_D - s_A$
+constants

Differentiating:

$$0 = 2v_B - 3v_A \quad ; \quad 0 = 2a_B - 3a_A$$

So $v_A = \frac{2}{3} v_B = \frac{2}{3}(3) = 2$ ft/sec

$a_A = \frac{2}{3} a_B = \frac{2}{3}(6) = 4$ ft/sec$^2$

$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$

$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$

The length of cable between A and C is

$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C$ + constants

$\Rightarrow 0 = 2v_B - v_A - v_C \; ; \; v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$

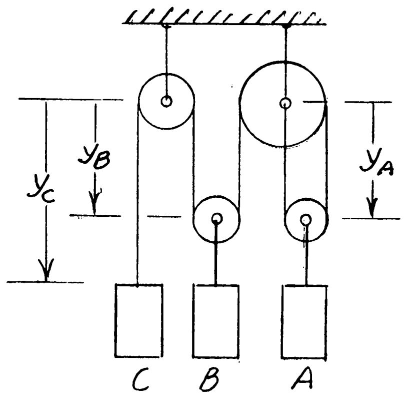
(All answers are quantities directed up incline.)

2/215 Length of cable $L = 2y_A + 2y_B + y_C + \text{const.}$

$$0 = 2\dot{y}_A + 2\dot{y}_B + \dot{y}_C$$

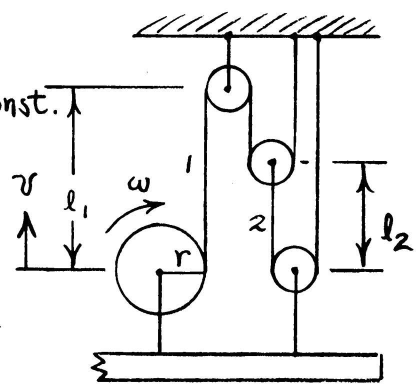
$$0 = 2\ddot{y}_A + 2\ddot{y}_B + \ddot{y}_C$$

so $\underline{2a_A + 2a_B + a_C = 0}$




2 degrees of freedom

2/216




Length $L_1 = \ell_1 + 2(\ell_1 - \ell_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 3\dot{\ell}_1 - 2\dot{\ell}_2$

Length $L_2 = \ell_2 + \ell_1 + \text{const.}$

$\dot{L}_2 = 0 = \dot{\ell}_2 + \dot{\ell}_1, \quad -\dot{\ell}_1 = \dot{\ell}_2$

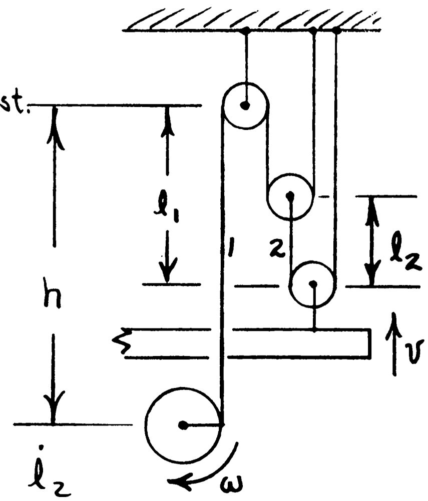
But $v = -\dot{\ell}_1$, so

$-r\omega = 3(-v) - 2v, \quad r\omega = 5v$

$v = \dfrac{r\omega}{5} = \dfrac{0.1(40)(2\pi/60)}{5} = 0.0838 \ \dfrac{m}{s}$

or $\underline{v = 83.8 \text{ mm/s}}$

2/217




Length $L_1 = h + 2(l_1 - l_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 0 + 2\dot{l}_1 - 2\dot{l}_2$

But $v = -\dot{l}_1$

So $-r\omega = -2v - 2\dot{l}_2$

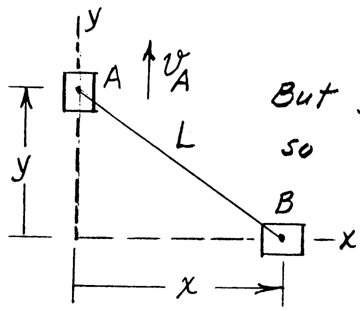
Length $L_2 = l_1 + l_2 + \text{const.}$

$\dot{L}_2 = 0 = \dot{l}_1 + \dot{l}_2$, $v = \dot{l}_2$

$\therefore r\omega = 2v + 2v$, $v = \dfrac{r\omega}{4} = \dfrac{0.1(40)(2\pi/60)}{4}$

$v = 0.1047$ m/s or $\underline{v = 104.7 \ \frac{mm}{s}}$

**2/218**




$$x^2 + y^2 = L^2 \; ; \quad x\dot{x} + y\dot{y} = 0$$

$$\& \quad \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

But $\dot{y} = v_A$, $\ddot{y} = 0$

so $$a_x = \ddot{x} = -\frac{\dot{x}^2 + \dot{y}^2}{x} = -\frac{\dot{y}^2 \frac{y^2}{x^2} + \dot{y}^2}{x}$$

$$a_x = -\frac{L^2}{x^3}\dot{y}^2 = -\frac{L^2 v_A^2}{(L^2 - y^2)^{3/2}}$$

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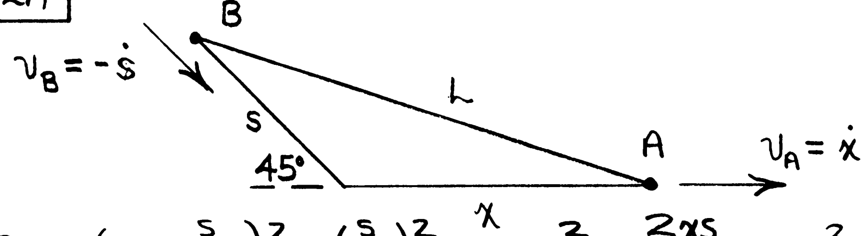



$$L^2 = \left(x + \frac{s}{\sqrt{2}}\right)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = x^2 + \frac{2xs}{\sqrt{2}} + s^2$$

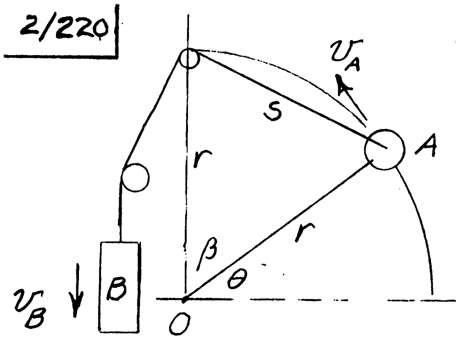
$$0 = 2x\dot{x} + \frac{2\dot{x}s}{\sqrt{2}} + \frac{2x\dot{s}}{\sqrt{2}} + 2s\dot{s} = 0$$

$$0 = x v_A + \frac{s}{\sqrt{2}} v_A - \frac{x}{\sqrt{2}} v_B - s v_B$$

$$v_B\left(s + \frac{x}{\sqrt{2}}\right) = v_A\left(x + \frac{s}{\sqrt{2}}\right) \text{ so } v_B = \frac{x + s/\sqrt{2}}{s + x/\sqrt{2}} v_A$$

$$\text{or } \underline{v_B = \frac{s + \sqrt{2}\,x}{x + \sqrt{2}\,s} v_A}$$

2/220




$v_B = -\dot{s}$ , $v_A = r\dot{\theta} = -r\dot{\beta}$

$s = 2r \sin \beta/2$

$\dot{s} = r\dot{\beta} \cos \beta/2$

so $-v_B = -v_A \cos \beta/2$

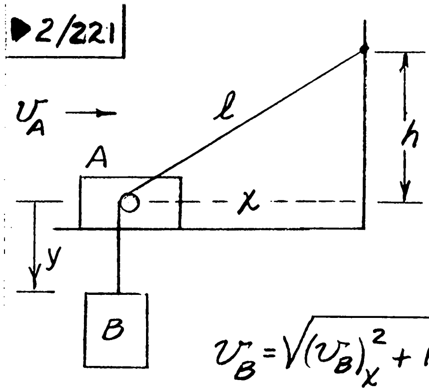
But $\cos \frac{\beta}{2} = \cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

Thus $v_B = v_A \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

$$v_A = \frac{v_B\sqrt{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}$$

**2/221**




$l^2 = x^2 + h^2$

$l\dot{l} = x\dot{x}$ , $\dot{y} = -\dot{l} = -\frac{x}{l}\dot{x}$

But $v_A = -\dot{x}$

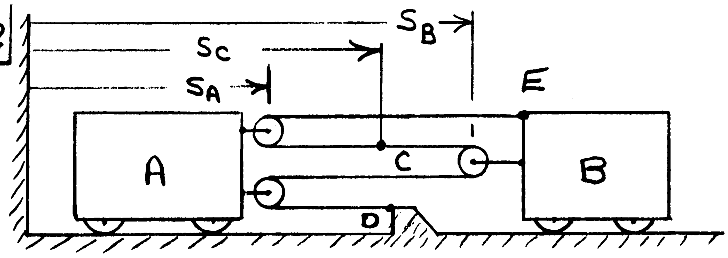
so $(v_B)_y = \dot{y} = \frac{x}{l} v_A$

$(v_B)_x = \dot{x} = -v_A$

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = v_A\sqrt{1 + \frac{x^2}{l^2}}$$

$$= v_A\sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}$$

▶2/222




Cable length $L = 3(s_B - s_A) + (s_D - s_A)$

$$0 = 3v_B - 4v_A \,, \quad 0 = 3a_B - 4a_A$$

$$v_A = \frac{3}{4} v_B = \frac{3}{4}(2) = 1.5 \text{ m/s}$$

$$a_A = \frac{3}{4} a_B = \frac{3}{4}(3) = 2.25 \text{ m/s}^2$$

$$v_{B/A} = v_B - v_A = 2 - (1.5) = \underline{0.5 \text{ m/s}}$$

$$a_{B/A} = a_B - a_A = 3 - (2.25) = \underline{0.75 \text{ m/s}^2}$$

Length of cable between points E and C:

$$L' = (s_B - s_A) + (s_C - s_A) + \text{constants}$$

$$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B$$

or $v_C = 2(1.5) - 2 = \underline{1 \text{ m/s}}$

(All answers are quantities directed to right)

$\boxed{2/223}$ $s = 8e^{-0.4t} - 6t + t^2$

$v = \frac{ds}{dt} = -3.2e^{-0.4t} - 6 + 2t$

$a = \frac{dv}{dt} = 1.28e^{-0.4t} + 2$

$a = 3 \text{ m/s}^2$ when $1.28e^{-0.4t} + 2 = 3$

$1.28e^{-0.4t} = 1$, $e^{-0.4t} = 0.781$

$-0.4t = -0.247$, $t = 0.617$ s

$v = -3.2e^{-0.4(0.617)} - 6 + 2(0.617)$

$= \underline{-7.27 \text{ m/s}}$

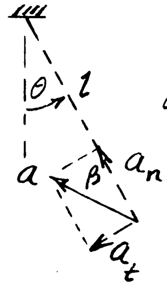
2/224 $+\uparrow$

$$v_y^2 = v_{y_0}^2 - 2g(y - y_0)$$

$$0^2 = v_0^2 - 2(32.2)(3)$$

$$\underline{v_0 = 13.90 \text{ ft/sec}}$$

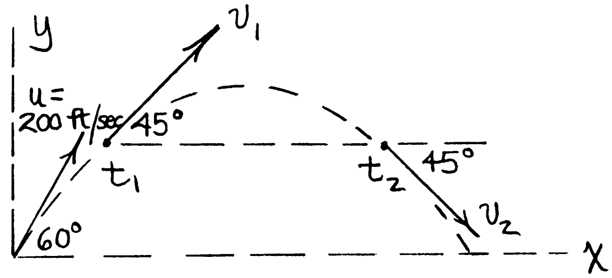
**2/225**




$$a_t = -l\ddot{\theta} = a \sin\beta \,, \quad \underline{\ddot{\theta} = -\frac{a}{l}\sin\beta}$$

$$a_n = l\dot{\theta}^2 = a\cos\beta \,, \quad \underline{\dot{\theta} = \pm\sqrt{\frac{a}{l}\cos\beta}}$$

2/226




$\dot{x} = u\cos\theta = 200\cos 60° = 100$ ft/sec

$\dot{y} = u\sin\theta - gt = 200\sin 60° - 32.2t = 173.2 - 32.2t$

At $t_1$: $\dot{x} = \dot{y}$: $100 = 173.2 - 32.2t_1$, $\underline{t_1 = 2.27 \text{ sec}}$

At $t_2$: $\dot{x} = -\dot{y}$: $100 = -173.2 + 32.2t_2$, $\underline{t_2 = 8.48 \text{ sec}}$

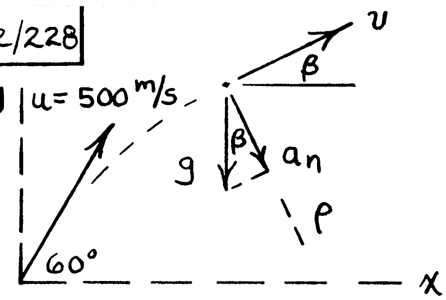
2/227 $\quad r = r_0 + b \sin \frac{2\pi t}{\tau}, \quad \dot{r} = \frac{2\pi}{\tau} b \cos \frac{2\pi t}{\tau}$

$$\ddot{r} = -\frac{4\pi^2}{\tau^2} b \sin \frac{2\pi t}{\tau}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{4\pi^2}{\tau^2} b \sin \frac{2\pi t}{\tau} - r\dot{\theta}^2 = 0$$

$$\Rightarrow \quad r = r_0 \frac{1}{1 + \left(\frac{\tau\dot{\theta}}{2\pi}\right)^2}$$

2/228




$v_x = 500 \cos 60° = 250 \text{ m/s}$

$v_y = v_{y_0} - gt = 500 \sin 60° - 9.81(30) = 138.7 \frac{m}{s}$

$v^2 = v_x^2 + v_y^2 = 250^2 + 138.7^2 = 81.7(10^3) \ m^2/s^2$

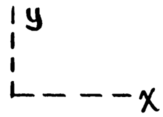
$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{138.7}{250} = 29.0°$

$a_n = g \cos\beta = 9.81 \cos 29.0° = 8.58 \text{ m/s}^2$

$a_n = v^2/\rho \Rightarrow \rho = \frac{v^2}{a_n} = \frac{81.7(10^3)}{8.58} = 9529 \text{ m}$

or $\underline{\rho = 9.53 \text{ km}}$

2/229




$$\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\underline{i} + 220\underline{i} = 172\underline{i} \ \frac{\text{km}}{\text{h}}$$

On descent : $\underline{v}_A = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j})$ km/h

$$\underline{v}_{A/C} = \underline{v}_A - \underline{v}_C = 172(\cos 10^\circ \underline{i} - \sin 10^\circ \underline{j}) - 30\underline{i}$$

$$= 139.4\underline{i} - 29.9\underline{j} \ \text{km/h}$$

$$\beta = \tan^{-1}\left(\frac{29.9}{139.4}\right) = \underline{12.09^\circ}$$

2/230 | $\ddot{y} = -g,\quad \dot{y} = u \sin 45° - gt,\quad y = ut \sin 45° - \frac{1}{2} g t^2$

$\ddot{x} = 0,\quad \dot{x} = u \cos 45°,\quad x = ut \cos 45°$

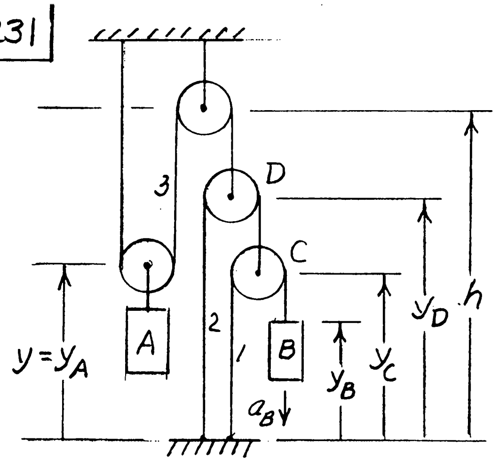
$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = 1 - \frac{gt}{u}\sqrt{2} = 1 - \frac{2gx}{u^2},\quad x = \frac{u^2}{2g}\left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \tan 20° = 0.3640,\quad x = \frac{(15)^2(10^6)^2}{(3600)^2(2)(9)(10^{-3})}(1 - 0.364) = 613 \text{ km}$$

$$t = \frac{x}{u \cos 45°} = \frac{613}{15000}\sqrt{2} = 0.0578 \text{ h} \quad \text{or} \quad \underline{3 \text{ min } 28 \text{ sec}}$$

$$h = y = \frac{15(10^3)(0.0578)}{\sqrt{2}} - \frac{9(10^{-3})}{2}(0.0578)^2(3600)^2 = \underline{418 \text{ km}}$$

2/231




$y = y_A = t^2/4$ m

$\dot{y}_A = t/2$ m/s

$a_A = \ddot{y}_A = 1/2$ m/s$^2$

one degree of freedom

Cable lengths

$L_1 = y_C + (y_C - y_B) + C_1 \qquad 0 = 2\ddot{y}_C - \ddot{y}_B$ where $a_B = -\ddot{y}_B$

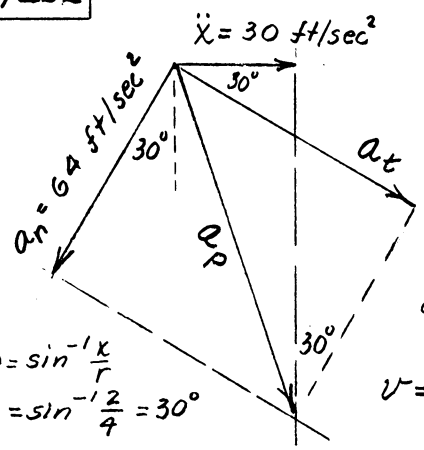
$L_2 = y_D + (y_D - y_C) + C_2 \qquad 0 = 2\ddot{y}_D - \ddot{y}_C$

$L_3 = 2(h - y_A) + h - y_D + C_3, \quad 0 = -2\ddot{y}_A - \ddot{y}_D$

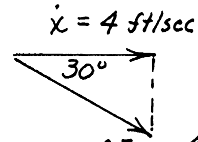
Eliminate $\ddot{y}_C$ & $\ddot{y}_D$ & get $\ddot{y}_B = -8\ddot{y}_A$ or $a_B = 8\ddot{y}_A = \underline{4 \text{ m/s}^2}$

(By inspection of pulley displacements
$-dy_B = 8\,dy_A$, so $a_B = 8a_A = 8(1/2) = 4$ m/s$^2$)

**2/232**




$\theta = \sin^{-1}\frac{x}{r}$
$= \sin^{-1}\frac{2}{4} = 30°$




$v = 4 / \frac{\sqrt{3}}{2} = \frac{8}{\sqrt{3}}$ ft/sec

$a_n = v^2/r = \left(\frac{8}{\sqrt{3}}\right)^2 / \frac{4}{12} = 64$ ft/sec²

$a_t = \frac{30}{\sqrt{3}/2} + \frac{64}{\sqrt{3}} = \frac{124}{\sqrt{3}} = 71.6$ ft/sec²

$v = r\dot{\theta}, \quad \dot{\theta} = \frac{v}{r} = \frac{8/\sqrt{3}}{4/12} = \underline{13.86 \text{ rad/sec}}$

$a_t = r\ddot{\theta}, \quad \ddot{\theta} = \frac{a_t}{r} = \frac{71.6}{4/12} = \underline{215 \text{ rad/sec}^2}$

2/233 $v_r = 3$ m/s, $v_\theta = 4$ m/s $\Big\}$ $v = 5$ m/s

$\tan\beta = 3/4$

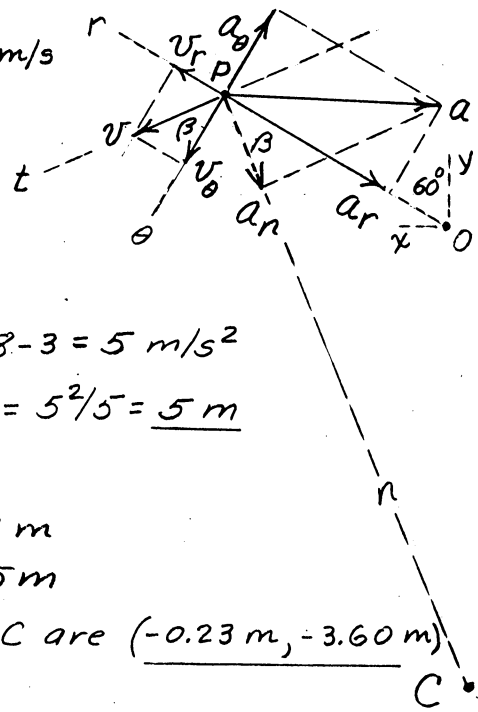
$a_r = -10$ m/s$^2$

$a_\theta = -5$ m/s$^2$

$a_n = |a_r|\cos\beta - |a_\theta|\sin\beta$

$= 10(4/5) - 5(3/5) = 8 - 3 = 5$ m/s$^2$

$a_n = v^2/\rho$, $\rho = v^2/a_n = 5^2/5 = \underline{5\text{ m}}$




$\overline{OP} = 2$ m

$\overline{CP} = 5$ m

x-y Coordinates of C are $\underline{(-0.23\text{ m}, -3.60\text{ m})}$

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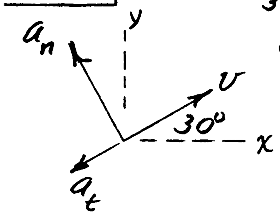


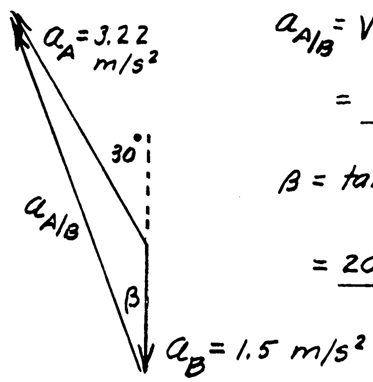

$v = \frac{1000}{3.6} = 278$ m/s , $a_t = \frac{15}{3.6} = 4.17$ m/s$^2$

$a_n = v^2/\rho = (278)^2/1500 = 51.4$ m/s$^2$

$\ddot{x} = -51.4 \sin 30° - 4.17 \cos 30° = \underline{-29.3 \text{ m/s}^2}$

$\ddot{y} = 51.4 \cos 30° - 4.17 \sin 30° = \underline{42.5 \text{ m/s}^2}$

2/235 $\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B}$ , $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3.22\ m/s^2$



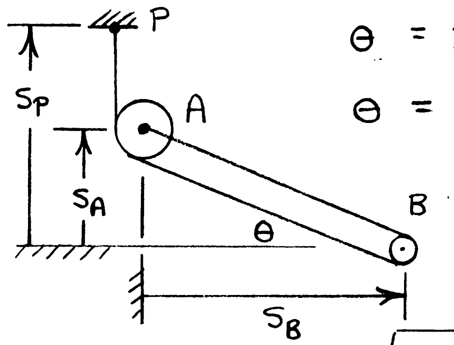

$a_{A/B} = \sqrt{(3.22\frac{\sqrt{3}}{2} + 1.5)^2 + (3.22\,[1/2])^2}$

$= \underline{4.58\ m/s^2}$

$\beta = \tan^{-1}\frac{1.608}{4.28} = \tan^{-1} 0.3752$

$= \underline{20.6°\ \text{west of north}}$

2/236




$\theta = 45°$ : $s_B/s_A = 1$

$\theta = 30°$ : $s_B/s_A = \sqrt{3}$

$\theta = 15°$ : $s_B/s_A = 3.73$

$$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$$

$$0 = -v_A + 2(s_A v_A + s_B v_B)\Big/\sqrt{s_A^2 + s_B^2}$$

$$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2\right]\frac{v_A}{2(s_B/s_A)}$$

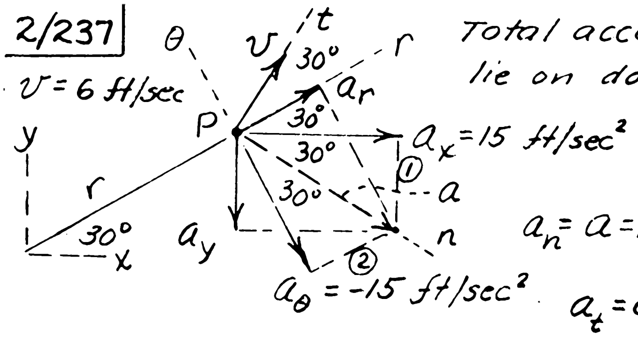
$\theta = 45°$ : $v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \text{ m/s}}$

$\theta = 30°$ : $v_B = \underline{0}$

$\theta = 15°$ : $v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \text{ m/s}}$

**2/237**

$v = 6$ ft/sec




Total acceleration must lie on dotted lines ① & ②

From 30-60 triangles,

$a_n = a = 15/\cos 30° = \underline{10\sqrt{3}\ \frac{ft}{sec^2}}$

$\underline{a_t = 0}$

$a_y = -15 \tan 30° = \underline{-5\sqrt{3}\ ft/sec^2}$, $a_r = 10\sqrt{3}\sin 30° = \underline{5\sqrt{3}\ ft/sec^2}$

$a_n = v^2/\rho$, $\rho = \frac{v^2}{a_n} = \frac{6^2}{10\sqrt{3}} = \underline{\frac{6\sqrt{3}}{5}\ ft}$

**2/238**

$$\begin{cases} x = 50 \text{ ft}, & \dot{x} = -10 \text{ ft/sec}, & \ddot{x} = -10 \text{ ft/sec}^2 \\ y = 25 \text{ ft}, & \dot{y} = 10 \text{ ft/sec}, & \ddot{y} = 5 \text{ ft/sec}^2 \end{cases}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = \underline{10\sqrt{2} \text{ ft/sec}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = \underline{11.18 \text{ ft/sec}^2}$$

$$\underline{e}_t = \underline{v}/v = (-10\underline{i} + 10\underline{j})/10\sqrt{2} = \underline{\tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j})}$$

$$a_t = \underline{a} \cdot \underline{e}_t = (-10\underline{i} + 5\underline{j}) \cdot \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{10.61 \text{ ft/sec}^2}$$

$$\underline{a}_t = a_t \underline{e}_t = 10.61 \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{-7.5\underline{i} + 7.5\underline{j} \text{ ft/sec}^2}$$

$$\underline{a}_n = \underline{a} - \underline{a}_t = (-10\underline{i} + 5\underline{j}) - (-7.5\underline{i} + 7.5\underline{j}) = \underline{-2.5(\underline{i} + \underline{j}) \text{ ft/sec}^2}$$

$$a_n = \sqrt{2.5^2 + 2.5^2} = \underline{3.54 \text{ ft/sec}^2}$$

$$\rho = v^2/a_n = (10\sqrt{2})^2/3.54 = \underline{56.6 \text{ ft}}$$

$$\underline{e}_n = \underline{a}_n/a_n = -2.5(\underline{i} + \underline{j})/3.54 = \underline{-\tfrac{\sqrt{2}}{2}(\underline{i} + \underline{j})}$$

$$\underline{e}_r = \underline{r}/r = 50\underline{i} + 25\underline{j} \Big/ \sqrt{50^2 + 25^2} = \underline{0.894\underline{i} + 0.447\underline{j}}$$

$$\underline{e}_\theta = \underline{e}_r \text{ rotated CCW } 90^\circ = \underline{-0.447\underline{i} + 0.894\underline{j}}$$

$$v_r = \underline{v} \cdot \underline{e}_r = (-10\underline{i} + 10\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-4.47 \text{ ft/sec}}$$

$$\underline{v}_r = v_r \underline{e}_r = -4.47(0.894\underline{i} + 0.447\underline{j}) = \underline{-4\underline{i} - 2\underline{j} \text{ ft/sec}}$$

$$v_\theta = \underline{v} \cdot \underline{e}_\theta = (-10\underline{i} + 10\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = \underline{13.42 \text{ ft/sec}}$$

$$\underline{v}_\theta = v_\theta \underline{e}_\theta = 13.42(-0.447\underline{i} + 0.894\underline{j}) = \underline{-6\underline{i} + 12\underline{j} \text{ ft/sec}}$$

$$a_r = \underline{a} \cdot \underline{e}_r = (-10\underline{i} + 5\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-6.71 \text{ ft/sec}^2}$$

$$\underline{a}_r = a_r \underline{e}_r = -6.71(0.894\underline{i} + 0.447\underline{j}) = \underline{-6\underline{i} - 3\underline{j} \text{ ft/sec}^2}$$

$$a_\theta = \underline{a} \cdot \underline{e}_\theta = (-10\underline{i} + 5\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = \underline{8.94 \text{ ft/sec}^2}$$

$$\underline{a}_\theta = a_\theta \underline{e}_\theta = 8.94(-0.447\underline{i} + 0.894\underline{j}) = \underline{-4\underline{i} + 8\underline{j} \quad \text{ft/sec}^2}$$

$$r = \sqrt{x^2+y^2} = \sqrt{50^2+25^2} = \underline{55.9 \text{ ft}}$$

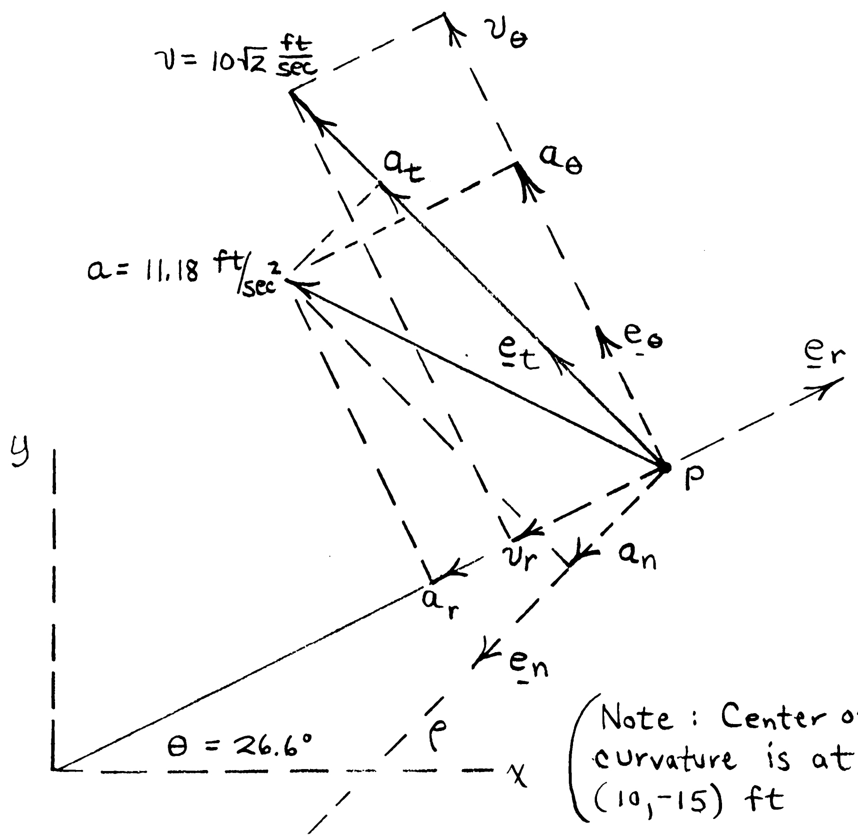
$$\dot{r} = v_r = \underline{-4.47 \text{ ft/sec}}$$

$$v_\theta = r\dot{\theta}, \quad \dot{\theta} = v_\theta / r = 13.42/55.9 = \underline{0.240 \text{ rad/sec}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9(0.240)^2$$
$$= \underline{-3.49 \text{ ft/sec}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta})$$
$$= \frac{1}{55.9}\left[8.94 - 2(-4.47)(0.240)\right] = \underline{0.1984 \text{ rad/sec}^2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6^\circ}$$




(Note : Center of curvature is at (10, −15) ft

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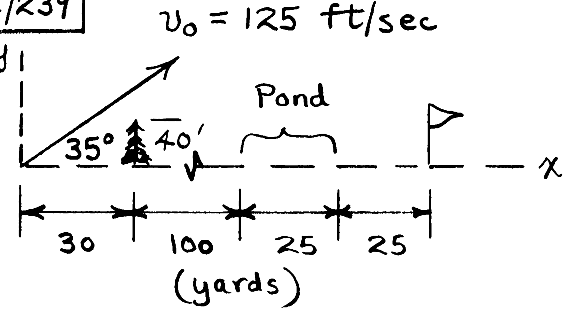



Time to tree: $x = x_0 + v_{x_0} t$ : $90 = 0 + 125 \cos 35° t$

$t = 0.879$ sec

Altitude : $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$

$y = 0 + 125 \sin 35° (0.879) - 16.1 (0.879)^2 = 50.6$ ft

So ball clears (slender) tree.

Flight time (y-eq.) : $0 = 0 + 125 \sin 35° t_f - 16.1 t_f^2$

$t_f = 0$ (launch time) or $t = 4.45$ sec (impact time)

Range (x-eq.) : $R = 0 + 125 \cos 35° (4.45)$

$= 456$ ft or $\underline{152.0 \text{ yd}}$

Ball lands in water hazard!

**▶2/240** $\theta = \theta_0 \cos\omega t$, $\dot\theta = -\theta_0\omega \sin\omega t$, $\ddot\theta = -\theta_0\omega^2 \cos pt$

$\dot\phi = K$, $\ddot\phi = 0$, $R = b$, $\dot R = \ddot R = 0$

From Eq. 2/19

$a_R = 0 - bK^2 - b\theta_0^2\omega^2 \sin^2\omega t \cos^2\phi$

$a_\theta = b\cos\phi\,(-\theta_0\omega^2\cos\omega t) - 2b(-\theta_0\omega \sin\omega t)K\sin\phi$

$a_\phi = 0 + b(\theta_0\omega \sin\omega t)^2 \sin\phi\cos\phi$

At A $\quad \cos\omega t = -1$, $\sin\omega t = 0$

$a_R = -bK^2$, $a_\theta = b\omega^2\theta_0\cos\phi$, $a_\phi = 0$

so $\underline{a = b\sqrt{K^4 + \omega^4\theta_0^2\cos^2\phi}}$

At B $\quad \cos\omega t = 0$, $\sin\omega t = 1$, $\phi = \pi/2$

so $\underline{a = bK\sqrt{K^2 + 4\omega^2\theta_0^2}}$

*2/241 $x_A = x_B$ : $0.16 \sin \frac{\pi t}{2} = 0.08 t$

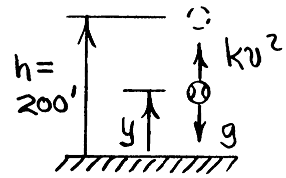
or $2 \sin \frac{\pi t}{2} - t = 0$

Solve numerically to obtain $\underline{t = 1.473\text{ s}}$

Then $x = 0.08(1.473) = \underline{0.1178\text{ m}}$

*2/242

$$a = v\frac{dv}{dy} = -g + kv^2$$




$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_h^y dy$$

$$\frac{1}{2k}\ln\left[-g+kv^2\right]_0^v = y\Big|_h^y$$

$$\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = y-h \Rightarrow v = \sqrt{\frac{g}{k}\left[1-e^{2k(y-h)}\right]}$$

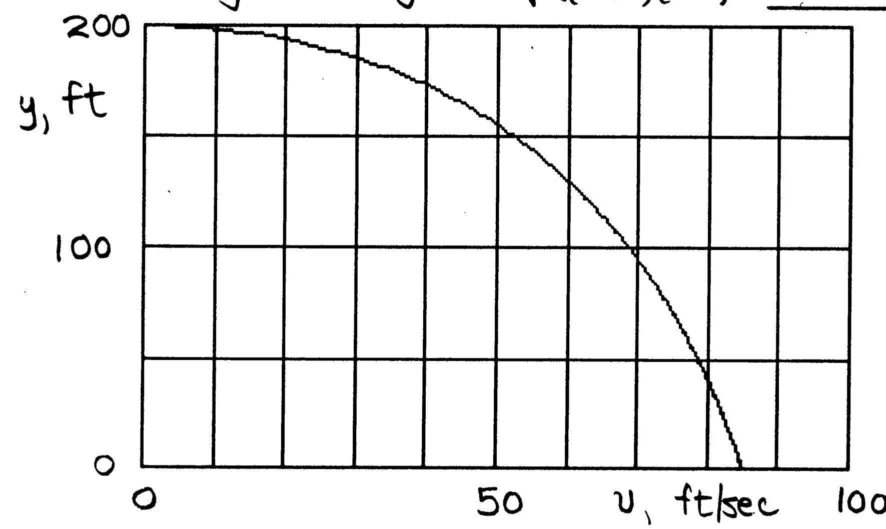
Given numbers : $85 = \sqrt{\frac{32.2}{k}\left[1-e^{2k(0-200)}\right]}$

Numerical solution : $\underline{k = 0.00323\ \text{ft}^{-1}}$

Terminal speed : $g = kv^2$ : $32.2 = 0.00323\,v_t^2$

$\underline{v_t = 99.8\ \text{ft/sec}}$

Without drag : $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = \underline{113.5\ \text{ft/sec}}$



***2/243** $r = 100 - 50\cos\theta$ , $\dot\theta = 2$ rad/s , $\ddot\theta = 0$

$\dot r = 50\dot\theta\sin\theta = 100\sin\theta$

$\ddot r = 100\dot\theta\cos\theta = 200\cos\theta$

$v_r = \dot r = 100\sin\theta$

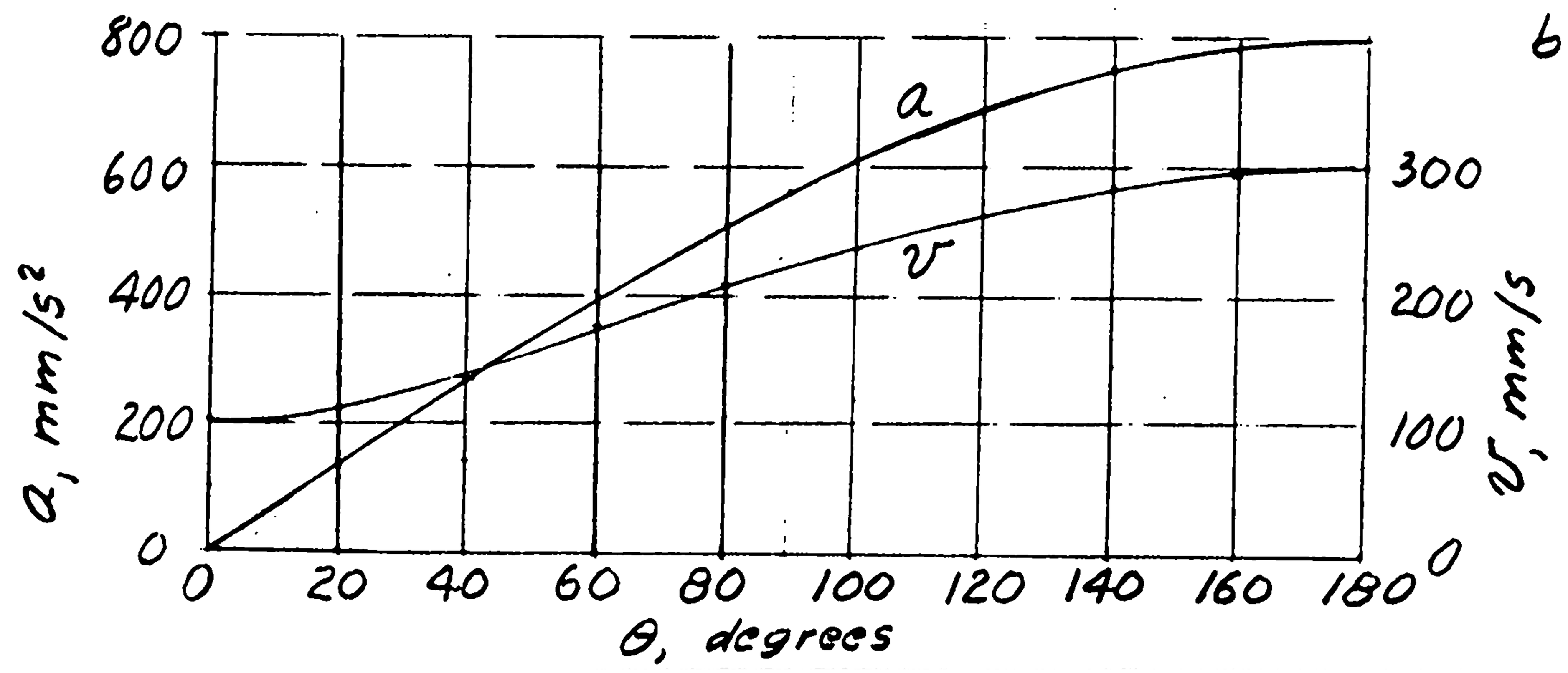
$v_\theta = r\dot\theta = (100 - 50\cos\theta)2 = 200 - 100\cos\theta$

$v = \sqrt{v_r^2 + v_\theta^2} = 100\sqrt{5 - 4\cos\theta}$

$a_r = \ddot r - r\dot\theta^2 = 200\cos\theta - 4(100 - 50\cos\theta) = 400(\cos\theta - 1)$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(100\sin\theta)2 = 400\sin\theta$

$a = \sqrt{a_r^2 + a_\theta^2} = 400\sqrt2\sqrt{1 - \cos\theta}$ [At $\theta = 0$, $a = a_r = 0$ since $\ddot r$ & $r\dot\theta^2$ cancel for $b = 2c$]



***2/244** $\ddot{\theta} = \dot{\theta}\dfrac{d\dot{\theta}}{d\theta} = \dfrac{g}{l}\cos\theta$

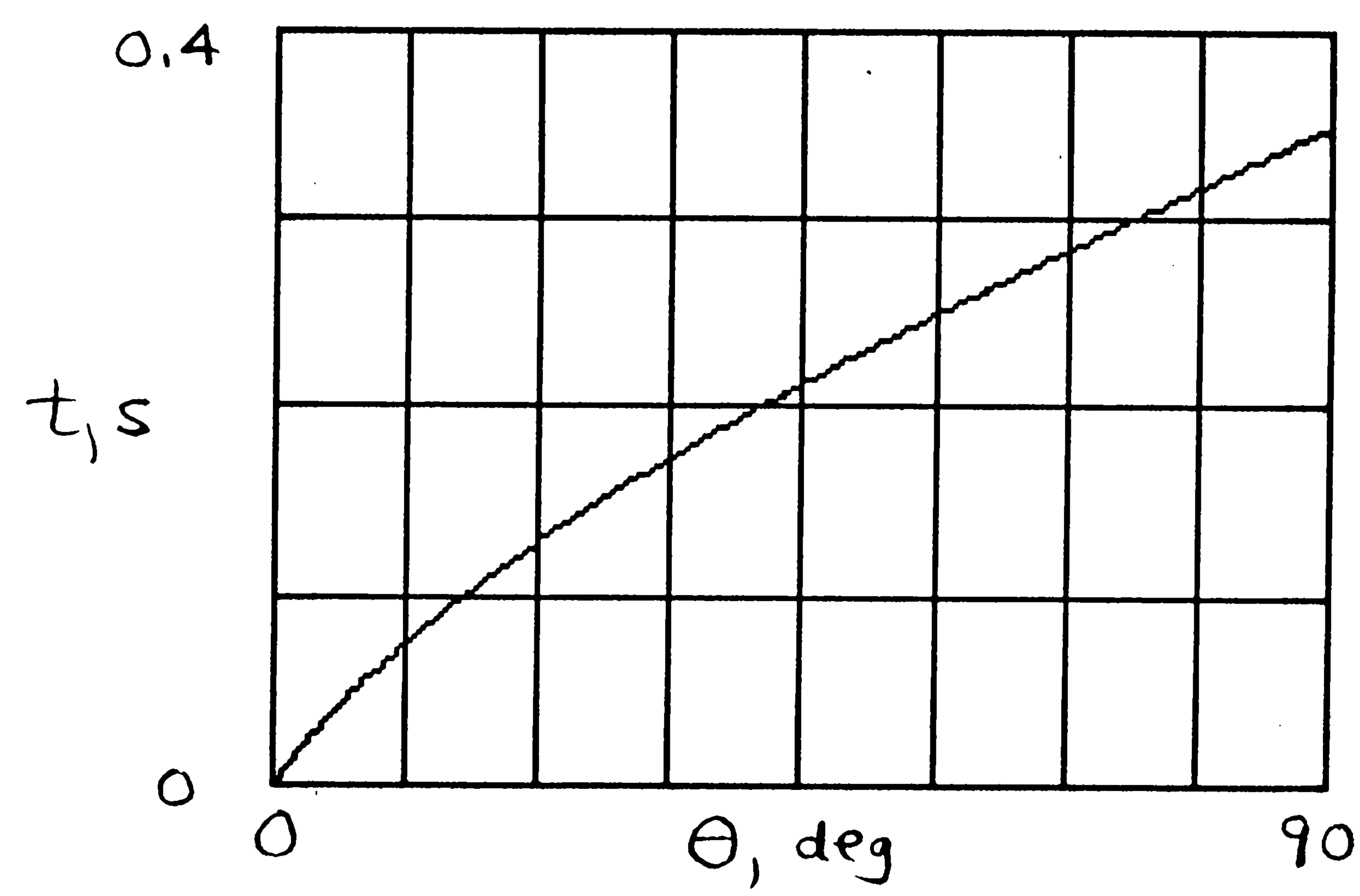
$$\int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \frac{g}{l}\int_0^{\theta} \cos\theta \, d\theta$$

$$\dot{\theta} = \left[\dot{\theta}_0^{\,2} + \frac{2g}{l}\sin\theta\right]^{1/2}$$

Then $\dot{\theta} = \dfrac{d\theta}{dt} = \left[\dot{\theta}_0^{\,2} + \dfrac{2g}{l}\sin\theta\right]^{1/2}$

$$t = \int_0^{\theta} \frac{d\theta}{\sqrt{\dot{\theta}_0^{\,2} + \frac{2g}{l}\sin\theta}}$$

With $\dot{\theta}_0 = 2$ rad/s, $l = 0.6$ m, $g = 9.81$, & $\theta = \frac{\pi}{2}$, a numerical integration yields $\underline{t' = 0.349 \text{ s}}$.



***2/245** Let $\omega_n = \sqrt{g/l}$ : $\begin{cases} \theta = \theta_0 \sin \omega_n t \\ \dot\theta = \theta_0 \omega_n \cos \omega_n t \\ \ddot\theta = -\theta_0 \omega_n^2 \sin \omega_n t \end{cases}$

$$a_t = l\ddot\theta = -l\theta_0\omega_n^2 \sin \omega_n t = -g\theta_0 \sin \omega_n t$$

$$a_n = l\dot\theta^2 = l\theta_0^2\omega_n^2 \cos^2 \omega_n t = g\theta_0^2 \cos^2 \omega_n t$$

$$a = \sqrt{a_t^2 + a_n^2} = g\theta_0\sqrt{\sin^2 \omega_n t + \theta_0^2 \cos^4 \omega_n t}$$

$a^2$ (and therefore $a$) is an extreme when

$$\frac{da^2}{d\theta} = 0 = g^2\theta_0^2\left[2\sin\omega_n t\,(\cos\omega_n t) + \theta_0^2\, 4\cos^3\omega_n t\,(-\sin\omega_n t)\right]$$

$$\Rightarrow \left[1 - 2\theta_0^2 \cos^2 \omega_n t\right] = 0$$

$$\left[1 - 2\left(\tfrac{\pi}{3}\right)^2 \cos^2 \omega_n t\right] = 0, \quad \omega_n t = 0.830 \text{ rad}$$

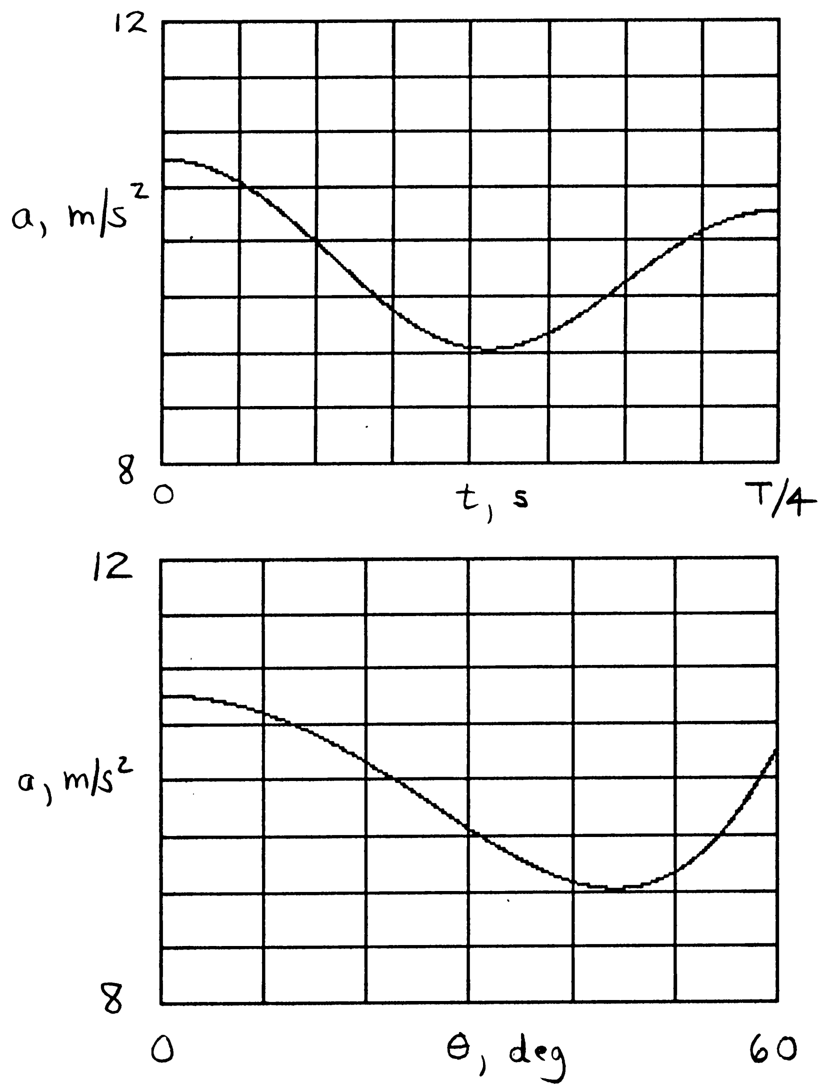
With $\omega_n = \sqrt{g/l} = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ s}^{-1}$, $t = 0.237$ s

$$\theta = \tfrac{\pi}{3}\sin(0.830) = 0.772 \text{ rad} \quad (44.3°)$$

As can be seen from the plots below, the above represents a minimum:

$a_{min} = 9.03 \text{ m/s}^2$ @ $\theta = 44.3°$ & $t = 0.237$ s

$a_{max} = 10.76 \text{ m/s}^2$ @ $\theta = 0$ & $t = 0$




Note : Period $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.50} = 1.794\,s$

$\frac{T}{4} = 0.449\,s$

***2/246** $\theta = \theta_0 \sin\frac{2\pi t}{\tau} = \frac{\pi}{2}\sin\pi t$, $\dot{\theta} = \frac{\pi^2}{2}\cos\pi t$, $\ddot{\theta} = -\frac{\pi^3}{2}\sin\pi t$

$a_n = l\dot{\theta}^2 = l\frac{\pi^4}{4}\cos^2\pi t$ , $a_t = l\ddot{\theta} = -l\frac{\pi^3}{2}\sin\pi t$

$$a = \sqrt{a_n^2 + a_t^2} = \frac{l\pi^3}{2}\sqrt{\frac{\pi^2}{4}\cos^4\pi t + \sin^2\pi t}$$

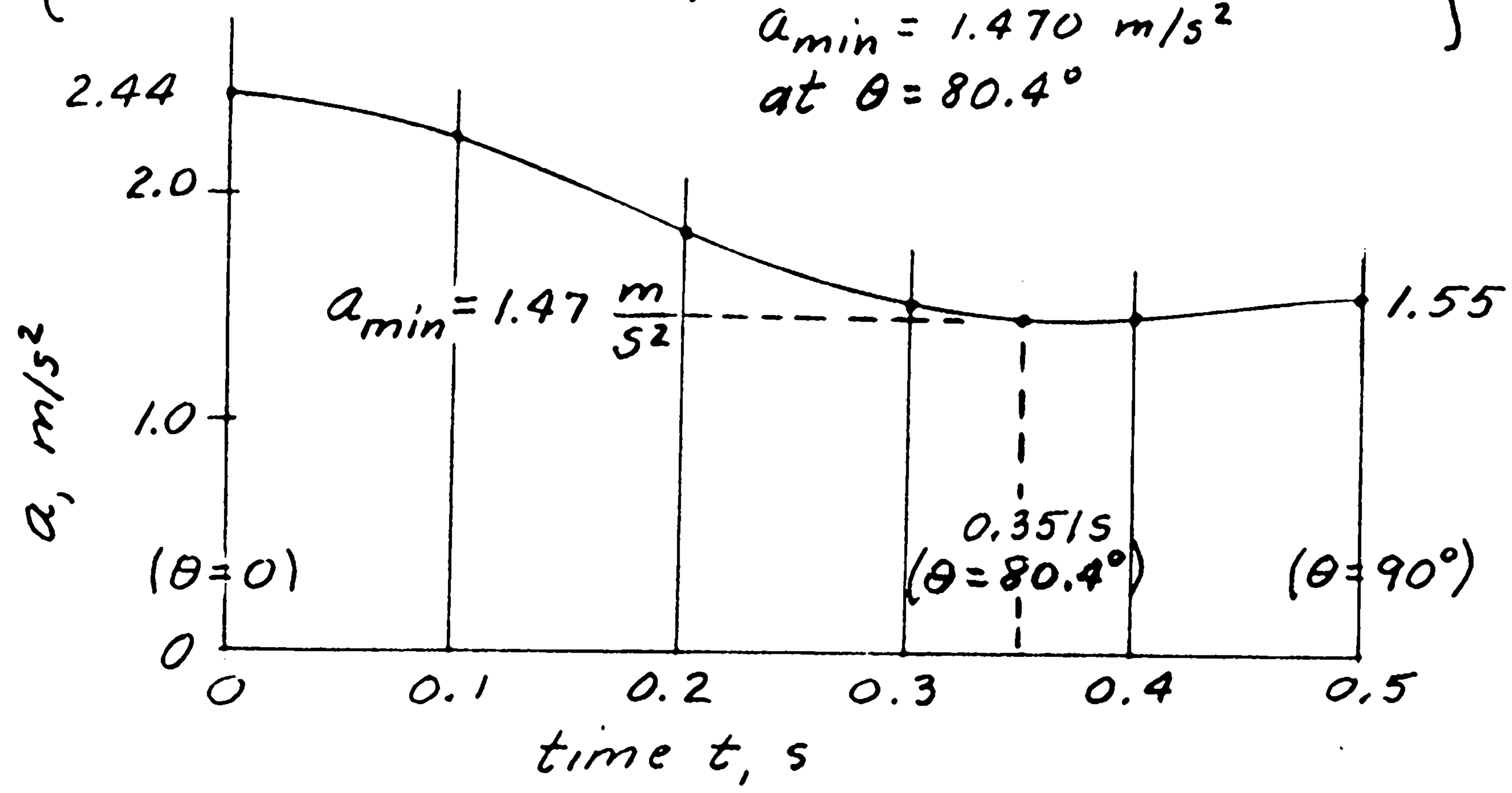
Program & solve for range $0 < t < 0.5$ s

$$\left\{ \frac{da^2}{dt} = \frac{l^2\pi^7}{8}\sin 2\pi t\,(2 - \pi^2\cos^2\pi t) = 0 \text{ for max \& min.} \right.$$

$\cos\pi t = \sqrt{2}/\pi$, $\pi t = 1.104$ rad, $t = 0.351$ s

$a_{min} = 1.470$ m/s$^2$

at $\theta = 80.4°$ $\Big\}$



*2/247

$$a = v\frac{dv}{dy} = -g - kv^2$$

$$\int_{v_0}^{0} \frac{v\,dv}{-g-kv^2} = \int_0^h dy$$

Let $u = g + kv^2$, $du = 2kv\,dv$: $\displaystyle\int_{v=v_0}^{v=0} -\frac{1}{2k}\frac{du}{u} = h$

$$h = -\frac{1}{2k}\ln\left(g+kv^2\right)\Big|_{v_0}^{0} = -\frac{1}{2k}\ln\left(\frac{g}{g+kv_0^2}\right)$$

$2kh = \ln\left(1+\dfrac{kv_0^2}{g}\right)$: $2(5280)k = \ln\left(1+\dfrac{2000^2 k}{32.2}\right)$

Solve numerically to obtain $\underline{k = 3.63\left(10^{-4}\right)\text{ ft}^{-1}}$

$\boxed{*2/248}$ $y = x^2/4$, $x$ & $y$ in inches ; $x = 4\sin 2t$, $t$ in seconds

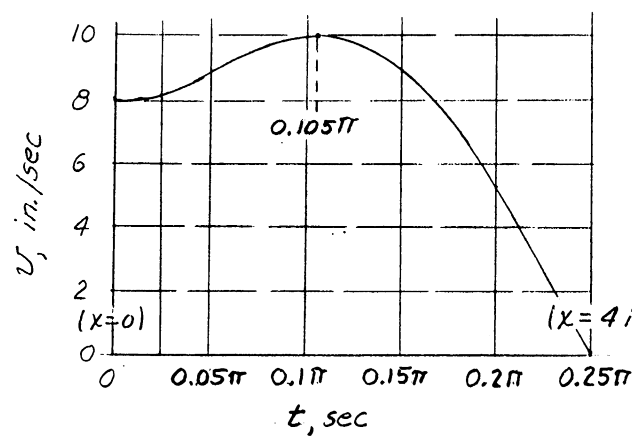
$$\dot{y} = \frac{x\dot{x}}{2} = 2\sin 2t\,(8\cos 2t) = 16\sin 2t\cos 2t \quad \text{in./sec}$$

$$\dot{x} = 8\cos 2t$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = 64\cos^2 2t + 256\sin^2 2t\cos^2 2t$$

$$= 64\cos^2 2t\,(1 + 4\sin^2 2t) \quad (\text{in./sec})^2$$

$$v = 8\cos 2t\sqrt{1 + 4\sin^2 2t}$$




$\frac{dv}{dt} = 0$ gives

$$1 - 2\sin^2 2t = 1/4$$

$$\sin 2t = \sqrt{3/8}$$

$$\cos 2t = \sqrt{5/8}$$

$$v = 8\sqrt{\tfrac{5}{8}}\sqrt{1 + 4(3/8)}$$

$$= \underline{10 \text{ in./sec}}$$

$$2t = 0.659 \text{ rad}$$

$$t = \underline{0.330 \text{ sec}}$$

$$(= 0.105\pi \text{ sec})$$

$$@\ \underline{x = 2.45 \text{ in.}}$$